Answers to Coursebook questions – Chapter H3

1 The rest energy is $E_0 = m_0c^2 = 1.67\times10^{-27}\times9\times10^{16} = 1.503\times10^{-10}\,\text{J} = 939\,\text{MeV}$.

2 $10m_0c^2 = \gamma m_0c^2 \Rightarrow \gamma = 10$. Hence $\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-0.01} = 0.995$.

3 $200 = \gamma\times135 \Rightarrow \gamma = 1.48$. Hence $\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{1.48^2}} = 0.738$.

4 **a** The kinetic energy is equal to the work done, i.e. 0.80 MeV.
The total energy is then $0.511 + 0.80 = 1.31\,\text{MeV}$.

b The gamma factor is $\gamma = \frac{1.31}{0.511} = 2.56$, hence $\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{2.56^2}} = 0.921$.

5 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$

and so $E = \gamma m_0c^2 = \frac{5}{3}\times938 = 1563 \approx 1600\,\text{MeV}$.

6 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.998^2}} = 15.82$.

The kinetic energy is then $E_K = (\gamma-1)m_0c^2 = 13900\,\text{MeV}$
and so $V = 13900\,\text{MV} = 13.9\,\text{GV}$.

7 It increases as measured by an observer with respect to whom the cube moves because the volume decreases: the length in the direction of motion contracts.

8 **a** The lifetime is $t = \frac{x}{v} = \frac{2.00\times10^3}{0.95\times3.0\times10^8} = 7.1\times10^{-6}\,\text{s}$.

b This observer measures the proper time interval between the events muon created and muon decays and so $\tau = \frac{t}{\gamma} = \frac{7.1\times10^{-6}}{\frac{1}{\sqrt{1-0.95^2}}} = 2.2\times10^{-6}\,\text{s}$.

9 The lifetime of the pion according to the lab is $t = \frac{20}{v}$

and also $t = \tau\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \times 2.6 \times 10^{-8}$. Hence, $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \times 2.6 \times 10^{-8} = \frac{20}{v}$.

This is best solved on the solver of your GDC.

Otherwise, $v^2 \times (2.6 \times 10^{-8})^2 = 20^2 (1 - \frac{v^2}{c^2})$.

This means $\frac{v^2}{c^2} \times 60.84 = 400(1 - \frac{v^2}{c^2}) \Rightarrow \frac{v^2}{c^2} = \frac{400}{460.84} \Rightarrow \frac{v}{c} = 0.931$

so that finally $v = 2.8 \times 10^8 \text{ m s}^{-1}$.

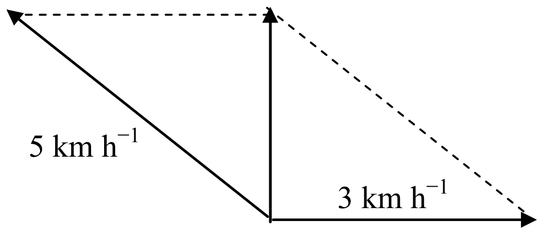
10 This problem and the next are useful in trying to understand the Michelson–Morley experiment.

a The speed down the stream is 6 km h^{-1} and so the time taken from A to B is $\frac{12}{6.0} = 2 \text{ h}$.

b The return trip is done at a speed of 2 km h^{-1} and so takes $\frac{12}{2.0} = 6 \text{ h}$.

c $2 \times \frac{12}{4.0} = 6 \text{ h}$.

11 A vector diagram of the velocities is as follows.



The speed with which the boat moves with respect to the barges is $\sqrt{5^2 - 3^2} = 4 \text{ km h}^{-1}$

and so the trip takes $\frac{12}{4.0} = 3 \text{ h}$.

Additional problem

A1 **a** Distinguish between rest and total energy.

b Explain why a particle such as the proton cannot be accelerated to the speed of light.

c A proton is accelerated from rest by a potential difference *V*. Draw sketch graphs to show the variation with *V* of the speed *v* of the proton after acceleration according to

i Newtonian mechanics

ii relativistic mechanics.

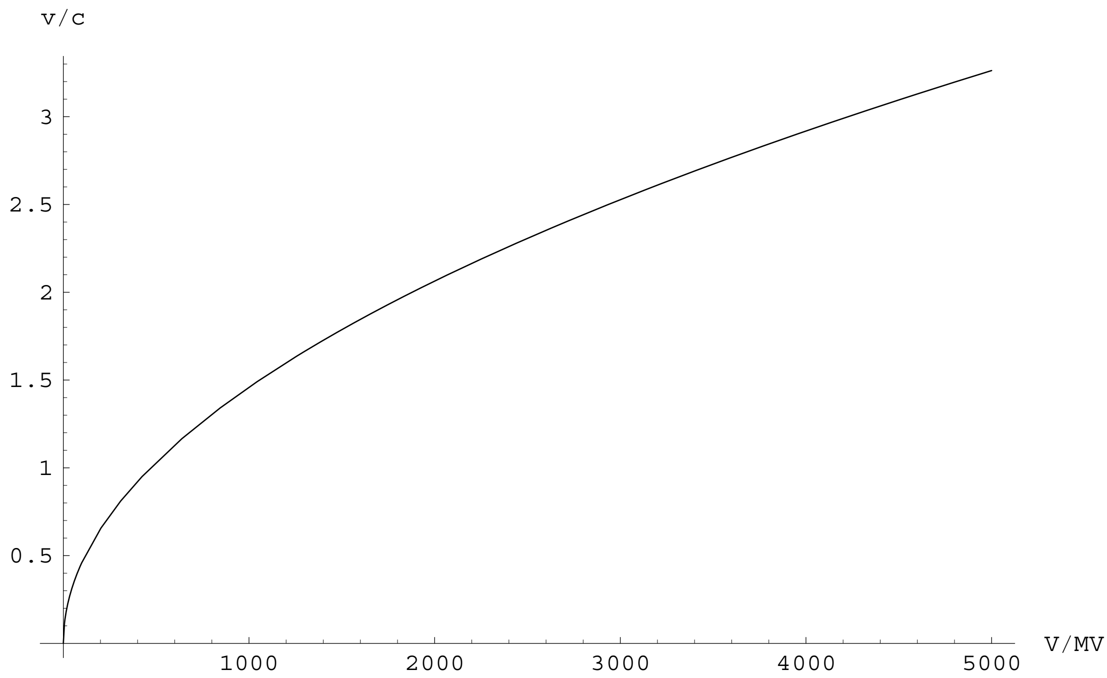
d Comment on the similarities and differences in your two sketches.

Answer

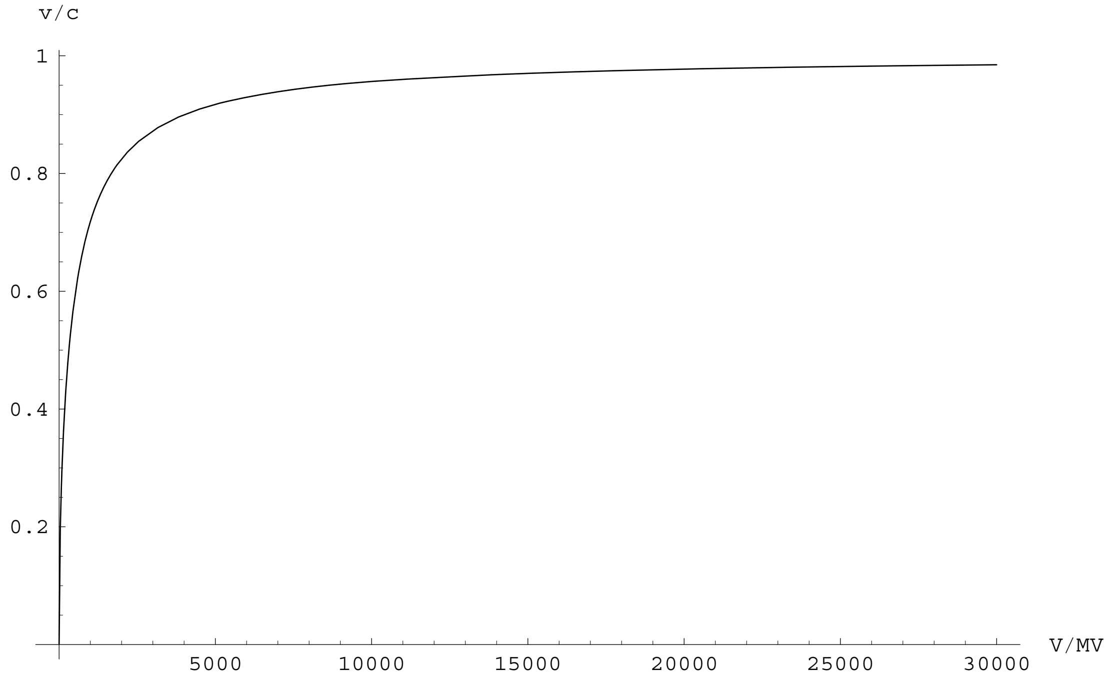
a Rest energy is the energy needed to create a particle. Total energy is the sum of the rest energy plus any other form of energy the particle has, e.g. kinetic energy.

b At the speed of light the proton would have an infinite total and kinetic energies and so it would require an infinite amount of work to accelerate it, hence this is impossible.

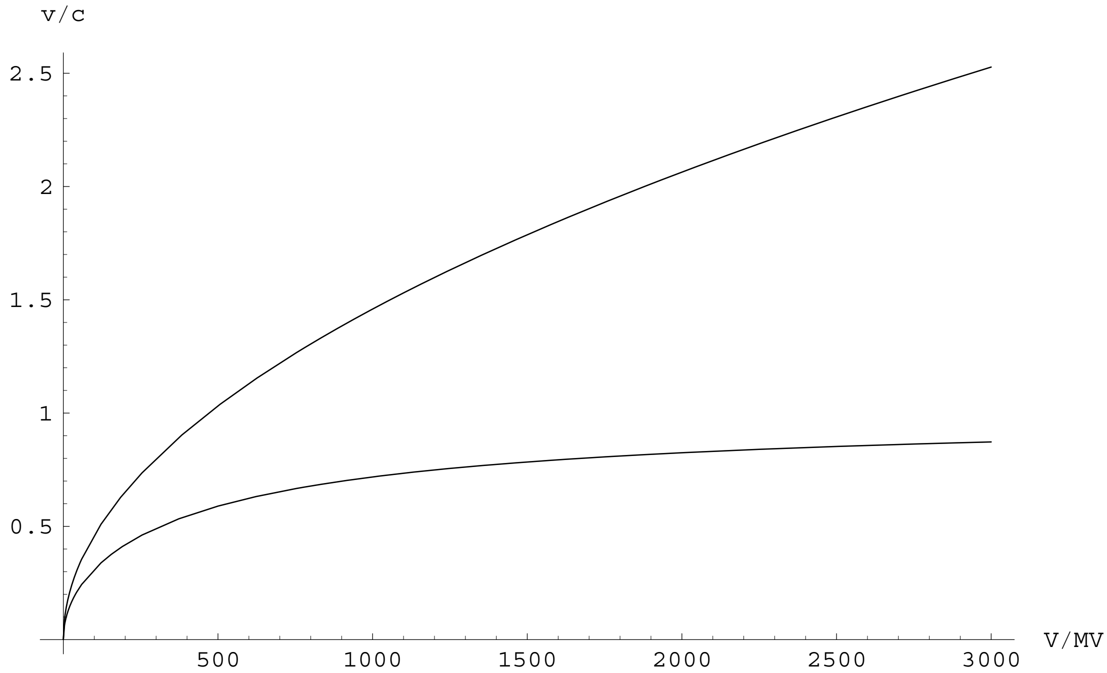
c The Newtonian graph is



The relativistic graph is



The two together are:



d The Newtonian graph shows the speed exceeding the speed of light, whereas in the relativistic graph the speed approaches the speed of light asymptotically. At low speeds, the two graphs are identical, since in the limit of small speeds (small compared to the speed of light) relativistic mechanics becomes identical to Newtonian mechanics.