Answers to Coursebook questions – Chapter H2

1 **a** The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.90^2}} = 2.294$.

The time interval of 5.0 min is a proper time interval for the earth observer and so for the Zenga invader the time interval is $\gamma \times 5.0 = 2.294 \times 5.0 = 11.47 \approx 11\,\text{min}$.

b 11 minutes, by exactly the same argument as in **a**.

2 **a** The interval of 5.0×10^{-8} s is a proper time interval.

The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.95^2}} = 3.20$, so the time interval for the observer in the lab is $\gamma \times 5.0 \times 10^{-8} = 3.20 \times 5.0 \times 10^{-8} = 1.6 \times 10^{-7}$ s.

b The distance travelled is $vt = 0.95 \times 3.0 \times 10^8 \times 1.6 \times 10^{-7} = 45.6 \approx 46\,\text{m}$.

3 **a** The time is $\dfrac{x}{v} = \dfrac{50\,\text{ly}}{0.995\text{c}} \approx 50.3\,\text{y}$.

b The time taken according to the spacecraft clocks will be the proper time and this is $\dfrac{50.3}{\gamma}$ y. The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.995^2}} \approx 10$. Hence the time is $\dfrac{50.3}{10} = 5.03\,\text{y}$. The students are just over 23 years old when they get to Vega.

4 The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.95^2}} = 3.20$.

The train observers measure the proper time interval.
So the ground observers measure $\gamma \times 1.0 = 3.20 \times 1.0 = 3.2\,\text{s}$.

5 **a** The time interval of 4.0 years is the proper time interval for the events: spacecraft leaves earth and spacecraft sends signal. The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.60^2}} = 1.25$. Hence according to the earth clocks the signal is sent after $\gamma \times 4.0 = 1.25 \times 4.0 = 5.0\,\text{y}$.

During this time the spacecraft has travelled a distance $x = vt = 0.60\text{c} \times 5.0 = 3.0\,\text{ly}$. This distance will be covered at the speed of light by the signal and so it will arrive on earth after 3.0 years according to earth.

b The earth is at a distance of $x' = vt' = 0.60\text{c} \times 4.0 = 2.40\,\text{ly}$.

In the time T it takes the signal to get to earth, the earth moved away a distance of $x = vt = 0.60\text{c} \times T$, so $\text{c}T = 0.60\text{c} \times T + 2.4 \Rightarrow T = \dfrac{2.4}{0.4} = 6.0\,\text{y}$.

c 3 ly as answered in **b**.

6 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.95^2}} = 3.20$. The 100 m is the length contracted distance and so the length at rest (the proper length) is $100\gamma = 3.20\times 100 = 320\,\text{m}$.

7 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$

a The distance of 8.0 ly is covered at the speed of light and so takes 8.0 years according to earth.

b The distance separating the spacecraft from the space station is $\frac{8.0}{5/3} = 4.8\,\text{ly}$ according to the spacecraft.
Therefore the space station covers this distance in $\frac{4.8\,\text{ly}}{0.80c} = 6.0\,\text{yr}$.

c The spacecraft is 8.0 ly away from earth (according to earth) when the signal is emitted. In the 8.0 years it takes the signal to arrive at earth the spacecraft moved an additional distance of $0.80\text{c}\times 8.0 = 6.4\,\text{ly}$. The reply signal will cover a distance c*T*,, where *T* is the required arrival time. Then (since spacecraft will travel a distance of $0.80\text{c}T$ in the meantime),
$\text{c}T = 8.0 + 6.4 + 0.80\text{c}T \Rightarrow T = 72\,\text{y}$.

d The time from the emission of the signal by the spacecraft and its reception is a proper time interval for the spacecraft.
According to earth the time interval between these two events is $8.0 + 72 = 80\,\text{y}$.
Hence the time for the spacecraft is $\frac{80}{5/3} = 48\,\text{y}$.

8 **a** The gamma factor is $\gamma = \frac{30}{28} = 1.07$.

The speed is then $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{\text{c}^2}}} \Rightarrow 1-\frac{v^2}{\text{c}^2} = \frac{1}{\gamma^2} \Rightarrow \frac{v}{\text{c}} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{1.07^2}} = 0.36$.

b Since the trains are identical the proper length of train A is 30 m, which B will measure to be length contracted to 28 m.

c 30 m, using the standard velocity addition formula.

9 The idea is to realize that this situation is analogous to the twin paradox situation. Observer Q is similar to the travelling twin, and hence his watch is slow when it comes back at B.

10 **a** According to the ground the light signal will take time T. In this time the rocket will move a distance vT closer to the mirror. Hence,

$$cT = D + (D - vT) \Rightarrow T = \frac{2D}{c+v} = \frac{4.8\times10^{12}}{1.90\times3.0\times10^{8}} = 8.42\times10^{3} \approx 8.4\times10^{3}\ \text{s}.$$

b The time for the rocket is the proper time interval since the signal is emitted and received at the same place. The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.90^2}} = 2.29$.

Hence $T' = \dfrac{8.42\times10^{3}}{2.29} = 3.7\times10^{3}\ \text{s}$.

11 **a** $u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$, $v = 0.6c$, $u = 0.8c$. Then $u' = \dfrac{0.2c}{1-0.8\times0.6} = 0.385c \approx 0.4c$.

b The answer is obviously $-0.385c$, but we can verify this from: $u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$ where now $v = 0.8c$, $u = 0.6c$ so that $u' = \dfrac{-0.2c}{1-0.8\times0.6} = -0.385c \approx -0.4c$.

12 **a** $u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$, $v = -0.6c$, $u = 0.8c$. Then $u' = \dfrac{-1.40c}{1-0.8\times(-0.6)} = -0.946c$.

b The answer is obviously 0.946c, but we can verify this from: $u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$, where now $v = 0.8c$, $u = -0.6c$ so that $u' = \dfrac{1.40c}{1-(-0.6)\times0.8} = 0.946c$.

13 Here we need to use $u = \dfrac{u'+v}{1+\dfrac{u'v}{c^2}}$ with $v = 0.60c$ and $u' = 0.70c$.

This gives $u = \dfrac{0.70c + 0.60c}{1+0.70\times0.60} = 0.915c$.

14 Here we need to use $u = \dfrac{u'+v}{1+\dfrac{u'v}{c^2}}$ with $v = -0.60c$ and $u' = 0.70c$.

This gives $u = \dfrac{0.70c + (-0.60c)}{1+0.70\times(-0.60)} = 0.172c$.

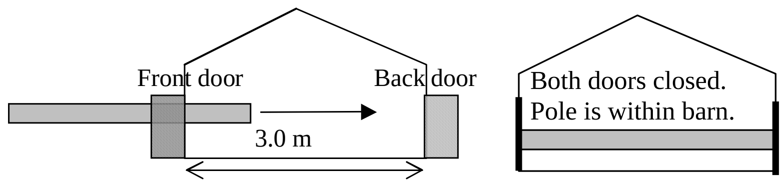
Additional problem

A1 A barn has proper length 3.0 m. A pole of proper length 5.0 m moves with speed 0.80c relative to the barn.

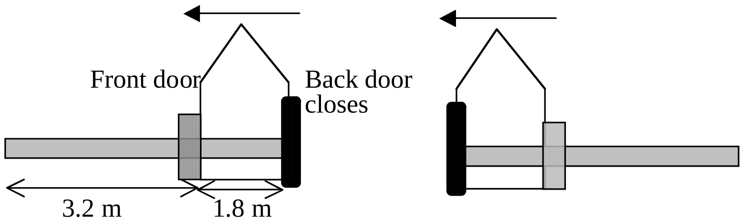
a Explain, by a suitable calculation, why the pole will fit completely within the barn according to the barn observers.

b The front and the back door of the barn close simultaneously (according to the barn observers) when the pole is completely within the barn. Identify the reason for which the two doors will not close simultaneously for the pole observer.

c The front door will in fact close **after** the back door, according to the rod observer. By drawing a diagram, according to the pole observer, the instant the front of the pole meets the back door, calculate how much later the front door closes. (Assume that the doors can be closed and then opened with zero time delay.)



Point of view of barn observer. The pole fits and does not crash into any door.



Point of view of pole observer. Doors do not close at the same time. The pole does not crash into a door.

Answer

a The gamma factor for a speed of 0.80c is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$. Hence the length of the pole is length contracted to $\frac{5.0}{5/3} = 3.0\,\text{m}$ and so fits within the barn.

b The closings of the doors are simultaneous for the barn observer and happen at **different points in space** so they will not be simultaneous for the pole observer.

c The length of the barn is $\frac{3.0}{5/3} = 1.8\,\text{m}$ (for the pole observer) and so there is a length of $5.0 - 1.8 = 3.2\,\text{m}$ outside of the barn.
The front door will close when it meets the left end of the pole and so it closes after a time given by $\frac{3.2}{0.80\text{c}} = 1.3\times10^{-8}\,\text{s}$.

