Mark scheme for Extension Worksheet – Option H, Worksheet 3

1 **a** The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$; and so

$p = \gamma m v = \frac{5}{3} \times 135 \times 0.80c\ \text{MeV}\,\text{c}^{-2} = 180\ \text{MeV}\,\text{c}^{-1}$ [2]

b $E = \gamma mc^2 = \frac{5}{3} \times (135\ \text{MeV}\,\text{c}^{-2})\, c^2 = 225\ \text{MeV}$ [1]

c Conservation of momentum says $p_Y - p_X = 180\ \text{MeV}\,\text{c}^{-1}$; The total energy of a photon is $E = pc$; so conservation of energy says $p_Y + p_X = 225\ \text{MeVc}^{-1}$; solving gives $p_Y = 202.5\ \text{MeVc}^{-1}$ and $p_X = 22.5\ \text{MeVc}^{-1}$ so the energies are simply $E_Y = 202.5\ \text{MeV}$ and $E_X = 22.5\ \text{MeV}$ [4]

2 **a** The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$; and so

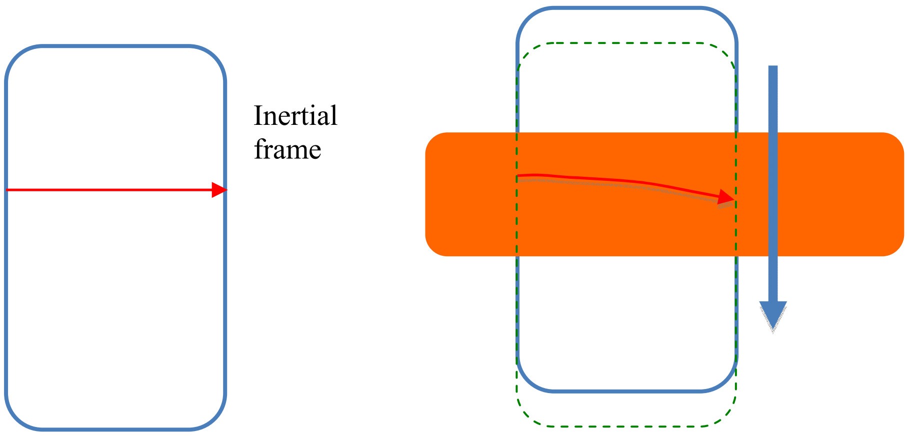
$E = \gamma mc^2 = \frac{5}{3} \times (600\ \text{MeV}\,\text{c}^{-2})\, c^2 = 1000\ \text{MeV}$ [2]

b The kinetic energy of one particle is
$E_K = \gamma mc^2 - mc^2 = 1000 - 600 = 400\ \text{MeV}$; and so the total is 800 MeV [2]

c The particle is produced at rest to conserve momentum and so by energy conservation $1000 + 1000\ \text{MeV} = Mc^2 \Rightarrow M = 2000\ \text{MeV}\,\text{c}^{-2}$ [1]

d Mass is not conserved in relativity; some mass may be converted to energy and vice versa in relativistic reactions. [2]

3 It is easiest to use the version of the equivalence principle that says that a freely falling frame of reference is equivalent to an inertial frame of reference far from all masses.



In the inertial frame of reference a ray of light directed parallel to the floor will hit the opposite wall at the same height; the same must happen in the freely falling frame; but since the frame has fallen the ray must hit lower as far outside observers are concerned , i.e. light bends. [3]

4 A box at rest on the surface of a planet is equivalent to a box accelerating in empty space; a ray of light is emitted from the base of the box and to observers outside, the top of the box is moving away from the light and so the frequency measured at the top will be less than that emitted by the Doppler effect; hence a ray of light rising in a gravitational field will have its frequency reduced. [3]

5 **a** The period is $\frac{1}{5.00\times10^{14}}=2.0\times10^{-15}\ \text{s}$; and so the number of full waves emitted in 2.00 μs is $\frac{2.0\times10^{-6}}{2.0\times10^{-15}}=10^{9}$ [2]

b Using $\Delta t=\frac{\Delta t_0}{\sqrt{1-\frac{R_S}{r}}}$ we get $\Delta t=\frac{2.0}{\sqrt{1-\frac{R_S}{2R_S}}}=\frac{2.0}{\sqrt{\frac{1}{2}}}=2.8\times10^{-6}\ \text{s}$ [2]

c In this interval the spacecraft receives 10^{9}; full waves and so the frequency is $\frac{10^{9}}{2.8\times10^{-6}}=3.6\times10^{14}\ \text{Hz}$, a smaller frequency as expected from gravitational redshift. [2]

6 $\Delta t=\frac{\Delta t_0}{\sqrt{1-\frac{R_S}{r}}}\Rightarrow10=\frac{5}{\sqrt{1-\frac{R_S}{r}}}$; $\Rightarrow1-\frac{R_S}{r}=\frac{1}{4}$; $\Rightarrow\frac{R_S}{r}=\frac{3}{4}\Rightarrow r=\frac{4R_S}{3}$ [3]

