Mark scheme for Extension Worksheet – Option H, Worksheet 2

1 *Part 1*

a R has velocity c and L has velocity $-c$ [1]

b For signal R the relative velocity is $c-v$ and for signal L it is $-c-v$, so the relative speed of L is just $c+v$ [2]

c These speeds appear to be the speeds of photons and are different from c; this is nothing to worry about as they are just relative velocities, not actual velocities of physical objects with respect to some observer and so relativity is not violated. [2]

d The R signal will cover a distance $\frac{L}{2}$ and will take time $\frac{L}{2(c-v)}$; whereas the L signal will take time $\frac{L}{2(c+v)}$; so the L signal arrives first by a time difference of $\frac{L}{2(c-v)}-\frac{L}{2(c+v)}=\frac{L}{2}\left(\frac{1}{c-v}-\frac{1}{c+v}\right)=\frac{L}{2}\frac{2v}{c^2-v^2}=\frac{Lv}{c^2-v^2}$ s [3]

e The proper length of the train L_0 is related to L through $L=\frac{L_0}{\gamma}$; and so the time difference of **d** is $\frac{Lv}{c^2-v^2}=\frac{1}{\gamma}\frac{L_0v}{c^2-v^2}=\frac{1}{\gamma c^2}\frac{L_0v}{1-\frac{v^2}{c^2}}=\frac{1}{\gamma c^2}\frac{L_0v}{1/\gamma^2}=\frac{\gamma L_0 v}{c^2}$ as required. [2]

Part 2

a L has velocity $-(c-v)$; and R has velocity $c+v$ [2]

b The relative velocity of L is $-(c-v)-v=-c$ and the relative velocity of R is $c+v-v=c$ [2]

c Since the relative speeds are the same the two signals will arrive at the same time according to the ground observer as well as the train observer, i.e. simultaneity is absolute in Galilean physics. [2]

2 **a** The time for the spaceship to arrive at the star is $\frac{4.0\text{ ly}}{0.60c}=6.67\text{ yr}$; the spaceship observers measure a proper time interval between leaving Earth and arriving at the star and so they measure a time of $\frac{6.67}{\gamma}=\frac{6.67}{\sqrt{1-0.60^2}}=5.3\text{ yr}$ [2]

b The mistake is that according to the spaceship observers the distance between Earth and the star is $\frac{4.0\text{ ly}}{\gamma}=\frac{4.0}{0.80}=3.2\text{ ly}$; and so the speed is $\frac{3.2\text{ ly}}{5.3\text{ y}}=0.60c$ as expected. [2]

3 **a** According to Earth the time is $\frac{60\text{ ly}}{0.80c} = 75\text{ yr}$ [1]

b The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$; since the rocket observers measure the proper time interval here, they measure $\frac{75}{5/3} = 45\text{ yr}$ [2]

c (i) According to the Earth this will take $\frac{60\text{ ly}}{c} = 60\text{ yr}$; (ii) According to rocket observers the time taken will be *t* where: in time *t* the signal covers distance ct, Earth moves away a distance $0.80ct$; and distance separating Earth and space station when signal is emitted is $\frac{60\text{ ly}}{\gamma} = \frac{60}{5/3} = 36\text{ ly}$; so
$ct = 0.80ct + 36 \Rightarrow 0.20ct = 36 \Rightarrow t = 180\text{ yr}$ [4]

4 The Michelson–Morley experiment was designed to measure the speed of the Earth relative to the ether; a key experimental feature involved the passage of light rays across two mutually normal lengths and observing their interference; by rotating by 90° the phase difference of the rays would change and a different interference pattern would be obtained if the speed of light was different in different directions in the ether. [3]

5 We need to use $u = \frac{u'+v}{1+\frac{u'v}{c^2}}$; so $u = \frac{(-0.40c)+0.80c}{1+\frac{(-0.40c)(0.80c)}{c^2}}$; giving $u = +0.59c$ [3]

6 From $E^2 = (mc^2)^2 + p^2c^2$ with $m = 0$ gives $p = \frac{E}{c}$; i.e. $p = 220\text{ MeV c}^{-1}$ [2]

7 We have that $\Delta E = 2.8\text{ MeV}$; hence $E - 0.511 = 2.8 \Rightarrow E = 3.31\text{ MeV}$; from $E^2 = (mc^2)^2 + p^2c^2$ we get $3.31^2 = (0.511)^2 + p^2c^2 \Rightarrow p = 3.27\text{ MeV c}^{-1}$; hence
$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{3.27\text{ MeV c}^{-1}} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3.27\times10^{6}\times1.6\times10^{-19}} = 3.8\times10^{-13}\text{ m}$ [4]

