Answers to Coursebook questions – Chapter 6.6

1

The distance of closest approach has been changed to 8.5×10^{-15} m.

By conservation of energy:

$$\frac{1}{2}mv^2=\frac{k(2e)(79e)}{d}\Rightarrow v=\sqrt{\frac{2k(2e)(79e)}{md}}$$

$$v=\sqrt{\frac{2\times9\times10^9(2\times1.6\times10^{-19})(79\times1.6\times10^{-19})}{6.4\times10^{-27}\times8.5\times10^{-15}}}$$

$v=3.7\times10^7$ m s^{-1}.

2 The idea is that since the nucleus is very massive it will not recoil.
Then at the point of closest separation the kinetic energy will be a minimum and will increase as the separation increases.

The potential energy is given by $E_P=\frac{kZe^2}{r}$ and so will be a maximum at the point of closet separation and will tend to zero as the separation increases.
These observations give the graphs in the answers in the textbook (see page 808 in *Physics for the IB Diploma*).

3 **a** The velocity selector ensures that ions of the given speed continue undeflected.
This means that the electric and magnetic forces on the ions cancel out.
Hence $qE=qvB\Rightarrow E=vB=2.0\times10^5\times0.15=3.0\times10^4$ N C^{-1}.

b From $qvB=\frac{mv^2}{r}$ we deduce that $r=\frac{mv}{qB}$.
We are given the molar mass and, since one mole contains Avogadro's number of molecules, we have that the mass is $\frac{19.992\times10^{-3}}{6.02\times10^{23}}=3.3\times10^{-26}$ kg.

Hence, $r=\frac{3.3\times10^{-26}\times2.0\times10^5}{1.6\times10^{-19}\times0.50}=0.083$ m.

c The radius for neon-22 will be $r=\frac{0.083\times21.99}{19.992}=0.091$ m.

4 They are electrically neutral and so do not suffer scatterings the way a charged particle would.

5 Conservation of momentum gives: $mv = mu + Mw$

and conservation of energy gives: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}Mw^2$.

From the first equation, $u = \frac{mv - Mw}{m} = v - \frac{Mw}{m}$ and so

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(v - \frac{Mw}{m}\right)^2 + \frac{1}{2}Mw^2.$$

Solving for the nucleus speed we get:

$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2 - Mwv + \frac{1}{2}\frac{M^2w^2}{m} + \frac{1}{2}Mw^2$$

$$0 = -2Mmwv + M^2w^2 + Mmw^2$$

$$0 = -2mwv + (M+m)w^2$$

This means that either $w = 0$, which is not realistic, or $w = \frac{2mv}{m+M}$ as required.

6 **a** We know that $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{3.00} = 0.231\ \text{s}^{-1}$.

b We start with $\frac{1}{100} \times 6.02 \times 10^{23} = 6.02 \times 10^{21}$ nuclei and so:

i $N = 6.02 \times 10^{21} \times e^{-0.231 \times 1} = 4.78 \times 10^{21}$

ii $N = 6.02 \times 10^{21} \times e^{-0.231 \times 2} = 3.79 \times 10^{21}$

iii $N = 6.02 \times 10^{21} \times e^{-0.231 \times 3} = 3.01 \times 10^{21}$

7 **a** The probability of decay within a half-life is always $\frac{1}{2}$.

b The probability that the nucleus will not decay after the passage of three half-lives is $\underbrace{\frac{1}{2}}_{\text{no decay}} \times \underbrace{\frac{1}{2}}_{\text{no decay}} \times \underbrace{\frac{1}{2}}_{\text{no decay}} = \frac{1}{8}$.

Hence the probability that the nucleus will decay some time within three half-lives is $1-\frac{1}{8}=\frac{7}{8}=0.875$.

c The probability of decay in any one half-life interval is 0.5.

More mathematically, we want to find $P(D|N)$, where we use the notation of conditional probability, and the events D and N stand for D = decay in the next half-life and N = no decay in the first 4 half-lives, respectively.

Then $P(D|N)=\frac{P(D\cap N)}{P(N)}$.

Now, $P(N)=\frac{1}{2^4}=\frac{1}{16}$ and $P(D\cap N)=\frac{1}{32}$. Hence $P(D|N)=\frac{1}{2}$.

8 The half-life is so long, so that what we are really asked to find is the initial activity of 1.0 g of pure radium.

We have that $A=\lambda N_0 e^{-\lambda t}$, so that the initial activity is λN_0.

A mass of 1.0 g of radium corresponds to $\frac{1.0}{226.005}=0.0044$ moles

and hence $N_0=0.0044\times 6.02\times 10^{23}=2.65\times 10^{21}$ nuclei.

Since $\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{1600\times 365\times 24\times 60\times 60}=1.37\times 10^{-11}\ \text{s}^{-1}$

we find an activity of $1.37\times 10^{-11}\times 2.65\times 10^{21}=3.6\times 10^{10}$ Bq.

9 The decay constant is $\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{12}=0.0578\ \text{d}^{-1}$

and so $A=\lambda N_0 e^{-\lambda t}=3.5\times e^{-0.0578\times 20}=1.1$ MBq.

10 The decay constant is $\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{6\times 24\times 60\times 60}=1.34\times 10^{-6}\ \text{s}^{-1}$.

From $A=\lambda N_0 e^{-\lambda t}$ we find

$0.50\times 10^{6}=1.34\times 10^{-6}\times N_0 e^{-1.34\times 10^{-6}\times 24\times 60\times 60}\Rightarrow N_0=4.2\times 10^{11}$.

11 After time t the number of uranium atoms remaining in the rocks is $N = N_0 e^{-\lambda t}$ and so the number that decayed (and hence eventually became lead) is $N - N_0 = N_0(1 - e^{-\lambda t})$.

Hence we have that $\dfrac{N_0(1-e^{-\lambda t})}{N_0 e^{-\lambda t}} = 0.80$.

This means that $1 - e^{-\lambda t} = 0.80e^{-\lambda t} \Rightarrow 1 = 1.80e^{-\lambda t} \Rightarrow e^{\lambda t} = 1.80$.
Hence $\lambda t = \ln(1.80) = 0.5878$.

Since $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{4.5 \times 10^9} = 1.54 \times 10^{-10}\ \text{y}^{-1}$ we find $t = \dfrac{0.5878}{1.54 \times 10^{-10}} = 3.8 \times 10^9\ \text{y}$.

12 The method of Q11 may be used but here, clearly, a ratio of 1 to 7 corresponds to three half-lives and so the age is about $t = 3 \times 1.37 \times 10^9 = 4.1 \times 10^9\ \text{y}$.

13 **a** In any experiment the detector will receive radiation from natural sources (the 'background') in addition to the radiation from the radioactive source used in the experiment. The background is always present and so it is registered even after the activity of the source goes to zero.

b The background activity of about 20 Bq has been subtracted from each of the measurements. From the second graph we obtain a half-life of just over 7 minutes.

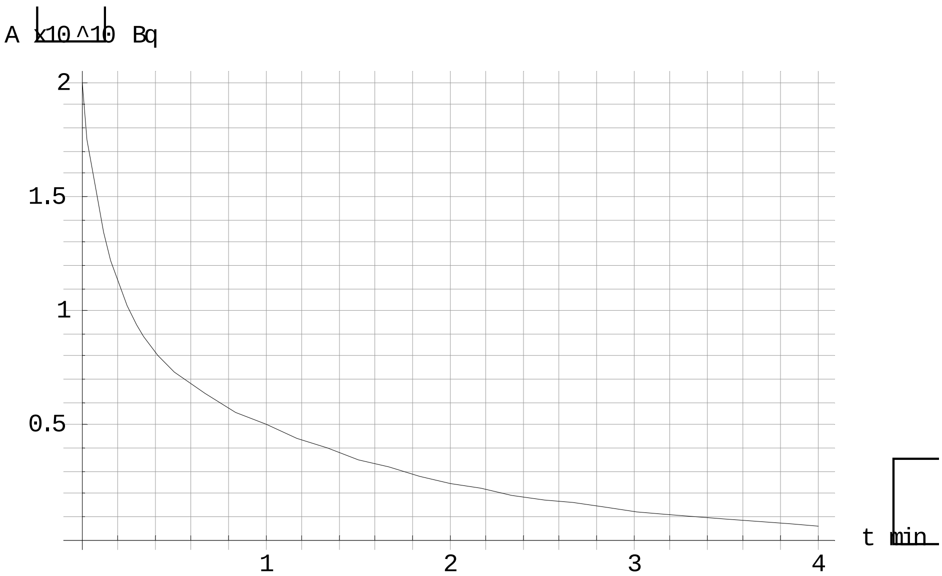
14 The activity is given by $A = \lambda N = \lambda N_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{T_{1/2}}$ is the decay constant.

a $\dfrac{A_A}{A_B} = \dfrac{\lambda_A N_{0A}}{\lambda_B N_{0B}} = \dfrac{3}{4} \times 1 = \dfrac{3}{4}$.

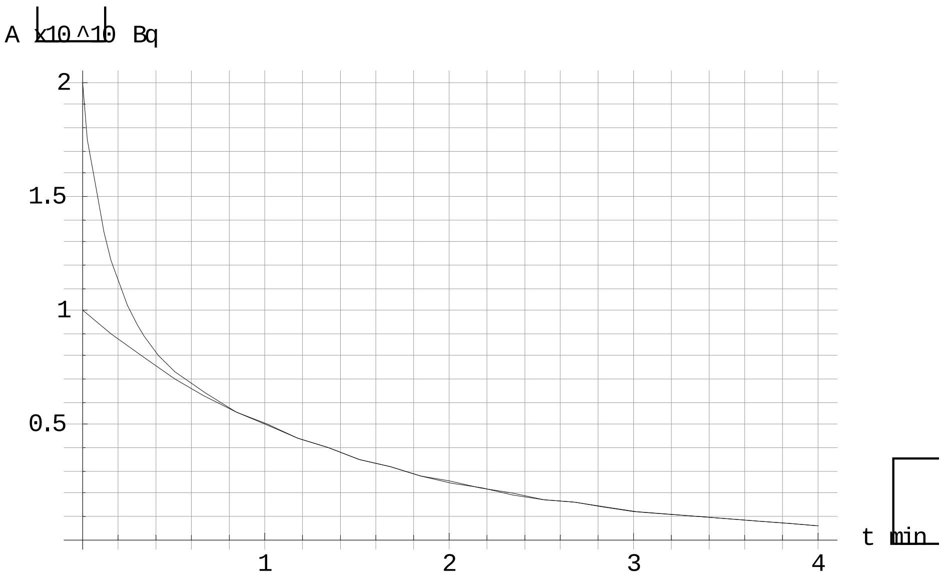
b $\dfrac{A_A}{A_B} = \dfrac{\lambda_A N_{0A} e^{-\lambda_A \times 4}}{\lambda_B N_{0B} e^{-\lambda_B \times 4}} = \dfrac{3}{4} \times \dfrac{e^{-\frac{\ln 2}{4} \times 4}}{e^{-\frac{\ln 2}{3} \times 4}} = 0.95$.

c $\dfrac{A_A}{A_B} = \dfrac{\lambda_A N_{0A} e^{-\lambda_A \times 12}}{\lambda_B N_{0B} e^{-\lambda_B \times 12}} = \dfrac{3}{4} \times \dfrac{e^{-\frac{\ln 2}{4} \times 12}}{e^{-\frac{\ln 2}{3} \times 12}} = 1.5$.

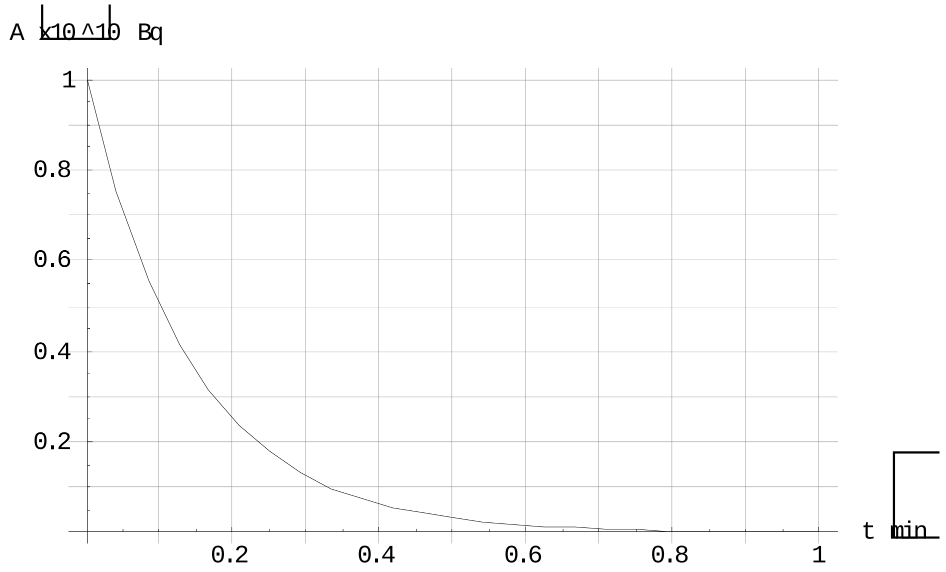
15 This is a very difficult question and many different possibilities must be considered. Essentially we must be able to determine from a graph of activity versus time the initial activities of the two isotopes and their respective half-lives. One possibility is represented by the following graph in which a short (S) and a long (L) half-life isotopes are present. The shape of the curve is not a pure exponential.



We see that after about 1 minute we have a smooth exponential curve which implies that one of the isotopes has essentially decayed away, leaving behind just one isotope. This is justified by estimating a half-life for times greater than 1 minute. We get consistently a half- life of 1 minute for the long half- life isotope. Extending the exponential curve smoothly backwards, we intercept the vertical axis at about 1×10^{10} Bq.



Thus the activity of isotope L is given by $A_L = 10^{10} \times 0.5^{t/1}$. This means that the initial activity of the other isotope is also 1×10^{10} Bq. Subtracting from the data points of the given graph the activity of this isotope we get the following graph:



This represents the decay of just isotope S. From this graph we find a half-life of about 0.1 minute.

Obviously, this analysis gets more complicated when the half-lives are not so different or when the initial activities are very different.

16 **a** If the mass (in grams) is m and the molar mass is μ, the number of moles of the radioactive isotope is $\frac{m}{\mu}$.

The initial number of nuclei is then $N_0 = \frac{m}{\mu} \times N_A$ since one mole contains Avogadro's number of molecules.

b The activity is $A = \lambda N = \lambda N_0 e^{-\lambda t} = \lambda \frac{m}{\mu} N_A e^{-\lambda t}$ and the initial activity is thus $A_0 = \lambda \frac{m}{\mu} N_A$. Measuring the initial activity then allows determination of the decay constant and hence the half-life from $\lambda = \frac{\ln 2}{T_{1/2}}$.

17 The radius of a nucleus of mass number A is $R = 1.2 \times A^{1/3} \times 10^{-15}$ m and its mass is $M = Am_n$ (here m_n is the mass of a nucleon).

The density is therefore $\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{A \times m_n}{\frac{4}{3}\pi (1.2 \times A^{1/3} \times 10^{-15})^3} = \frac{m_n}{\frac{4}{3}\pi (1.2 \times 10^{-15})^3}$

and so is independent of A.

18 **a** The force becomes negligible at separations of about 1.5×10^{-15} m.

b From the graph this is about 4 kN.

c $$F=\frac{9\times10^{9}\times\left(1.6\times10^{-19}\right)^{2}}{\left(1.0\times10^{-15}\right)^{2}}=230\text{ N}.$$

d $$F=\frac{6.67\times10^{-11}\times\left(1.67\times10^{-27}\right)^{2}}{\left(1.0\times10^{-15}\right)^{2}}=2\times10^{-34}\text{ N}.$$

e $$\frac{F_e}{F_g}=\frac{230}{2\times10^{-34}}\approx10^{36}.$$

19 Generally, this process takes place in heavy atoms and the captured electron is taken from the inner shell of electrons in the atom. This means that there will follow a transition of an atomic electron down to the ground state with the emission of a (typically X-ray) photon. Another way of telling is that the atom after the electron capture has changed atomic number, i.e. it is a different element.

20 **a** $${}^{226}_{88}\text{Ra}\rightarrow{}^{226}_{88}\text{Ra}+{}^{0}_{0}\gamma$$

b $$hf=\frac{hc}{\lambda}=\Delta E\Rightarrow\lambda=\frac{hc}{\Delta E}.$$

Hence $$\lambda=\frac{6.63\times10^{-34}\times3.0\times10^{8}}{0.0678\times10^{6}\times1.6\times10^{-19}}=1.83\times10^{-11}\text{ m}.$$

