Answers to Coursebook questions – Chapter 6.5

1 **a** Electrons in atoms exist in states with specific energy. A state with a specific energy corresponds to an energy level.

b An electron can change its state by moving from a state of high energy to a state of lower energy by emitting a photon of energy equal to the difference in energy of the levels involved in the transition. If an electron absorbs energy equal to the difference of energy levels it can also make a transition to a higher energy level.

c The set of photon wavelengths that can be emitted or absorbed by the atoms of an element.

2 The emitted photons are not necessarily in the same direction as that of the incident beam. Hence an observer looking along the beam direction will receive very few of the re-emitted photons.

3 The energies, in eV, of the hydrogen atom electron are found from $-\frac{13.6}{n^2}$ and so form the set $\{-13.6, -3.4, 1.51, 0.85, ...\}$.
The difference between excited levels and the ground state are $\{10.2, 12.1, 12.8, ...\}$.
Thus an electron with energy 11.5 eV can give 10.2 eV of its energy to a ground state electron that will make a transition to the level $n = 2$ and rebound with a kinetic energy $11.5 - 10.2 = 1.3$ eV.
Of course, the electron may lose no energy to the atom, in which case it will have an elastic collision moving away with the same energy as the original, i.e. 11.5 eV.

4 **a** The existence of absorption and emission spectra (see page 401 in *Physics for the IB Diploma*).

b We list the energy differences between the ground sate and the excited states:
$\Delta E_{12} = 10.2$ eV, $\Delta E_{13} = 12.1$ eV, $\Delta E_{14} = 12.8$ eV, $\Delta E_{15} = 13.1$ eV, $\Delta E_{16} = 13.2$ eV, $\Delta E_{17} = 13.3$ eV. Hence:

i not enough energy for an excitation,

ii the electron can reach $n = 4$ and

iii the electron can reach $n = 6$.

5 **a** This is the energy that must be supplied to an atom so that an electron can be ejected from the atom.

b The energy in the $n = 3$ level is $E_3 = -\frac{13.6}{3^2} = -1.51$ eV and this is the ionization energy for this level.

6 **a** The smallest wavelength corresponds to the largest energy difference, and this theoretically is 13.6 eV.

Hence $E = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3.0\times10^{8}}{13.6\times1.6\times10^{-19}} = 9.1\times10^{-8}$ m.

b The kinetic energy of the electron must be at least 13.6 eV

i.e. $\frac{1}{2}mv^2 = E_K \Rightarrow v = \sqrt{\frac{2\times13.6\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 2.2\times10^{6}\ \text{m s}^{-1}$.

7 **a** From $E_K = \frac{p^2}{2m}$ we find

$p = \sqrt{2mE_K} = \sqrt{2\times1.67\times10^{-27}\times200\times10^{6}\times1.6\times10^{-19}} = 3.27\times10^{-19}$ N s.

Hence $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{3.27\times10^{-19}} = 2.0\times10^{-15}$ m.

b The total energy of the electron in the state $n = 2$ is $E = \frac{-13.6}{2^2} = -5.44\times10^{-19}$ J.

The kinetic energy of the electron is the negative of the total energy and so $E_K = +5.44\times10^{-19}$ J.

Since $E_K = \frac{p^2}{2m}$ we find

$p = \sqrt{2mE_K} = \sqrt{2\times9.1\times10^{-31}\times5.44\times10^{-19}} = 9.95\times10^{-25}$ N s.

Hence $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{9.95\times10^{-25}} = 6.66\times10^{-10} \approx 6.7\times10^{-10}$ m.

8 We may take the uncertainty in the electron's position to be $\Delta x \approx 1\times10^{-10}$ m, the 'size' of the atom.

Then $\Delta p \geq \frac{h}{4\pi\Delta x} = \frac{6.63\times10^{-34}}{4\pi\times1\times10^{-10}} = 5.27\times10^{-25}$ N s.

The corresponding kinetic energy is then of order

$E_K = \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m} = \frac{(5.27\times10^{-25})^2}{2\times9.1\times10^{-31}} = 1.53\times10^{-19}\ \text{J} = \frac{1.53\times10^{-19}}{1.6\times10^{-19}} = 0.96 \approx 1\ \text{eV}$,

which is the correct order of magnitude.

9 **a** The wavelength will be given by $\lambda = \frac{2L}{n}$ and, for the fundamental, $\lambda = 2L = 2\times10^{-15}$ m.

b

$$\Delta p \geq \frac{h}{4\pi\Delta x} = \frac{6.63\times10^{-34}}{4\pi\times1\times10^{-15}} = 5.27\times10^{-20}\text{ N s.}$$

$$E_K = \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m} = \frac{(5.27\times10^{-20})^2}{2\times9.1\times10^{-31}} = 1.53\times10^{-9}\text{ J} = \frac{1.53\times10^{-9}}{1.6\times10^{-19}} \approx 10^{10}\text{ eV} = 10^4\text{ MeV.}$$

c This is far larger than the binding energy of a nucleus and so the electron would rip the nucleus apart. The electron cannot be confined within a nucleus.

10 The essential difference is that in the case of the hydrogen atom the spectral lines get closer together as the wavelength decreases. The limiting wavelength, i.e. the smallest wavelength that can be emitted by a hydrogen atom, corresponds to an energy level of 13.6 eV, i.e. $\frac{hc}{\lambda} = 13.6\times1.6\times10^{-19} \Rightarrow \lambda = \frac{6.63\times10^{-34}\times3.0\times10^{8}}{13.6\times1.6\times10^{-19}} = 9.1\times10^{-8}$ m.
For the 'electron in the box' model the spectral lines also tend to come closer together as the wavelength decreases, but the limiting wavelength is 0 since the energy differences become large without limit.

11 **a** There is a wave associated with every moving particle, of wavelength equal to Planck's constant divided by the momentum of the particle.

b The kinetic energy of the electron will be $E_K = qV$,

and so $\frac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV} = 1.21\times10^{-24}$ N s.

Then $\lambda = \frac{6.63\times10^{-34}}{1.21\times10^{-24}} = 5.5\times10^{-10}$ m.

c Precise knowledge of the wavelength implies precise knowledge of the momentum. By the uncertainty principle the uncertainty in position must be large.

12 **a** As the opening decreases, there will be more and more diffraction and so the beam will not be thin – it will spread.

b To reduce diffraction, the wavelength must be as small as possible (and smaller than *d*). This requires fast electrons.

13 The de Broglie wavelength of the tennis ball is

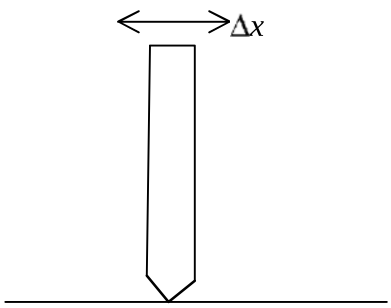
$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{6} = 1 \times 10^{-34}\ \text{m}.$$

The tennis ball wave will diffract through the opening.
The angle at which the first diffraction minimum occurs is of order

$$\theta_D = \frac{\lambda}{b} = \frac{1 \times 10^{-34}}{1} = 1 \times 10^{-34}\ \text{rad}.$$

The angle is insignificantly small. The tennis ball will move on a straight line without any deviation.

14 There will always be an uncertainty Δx in the position of the top of the pencil and so there will be a corresponding uncertainty in momentum.
Hence the top of the pencil will have to move and hence the pencil will fall.



15 **a** The top graph allows precise determination of the wavelength and hence the momentum. The uncertainty in momentum will then be small.

b The bottom diagram shows that the probability of finding the particle is large within a small area of space.

