Answers to Coursebook questions – Chapter 6.3

1 Note that the problem has given an atomic mass for nickel and we need the nuclear mass. Hence we must subtract the electron masses. The mass defect is

$$\begin{aligned}\delta &= 28m_p + (62-28)m_n - (M_{Ni} - Zm_e)\\ &= 28\times 1.007\,276 + 34\times 1.008\,665 - (61.928\,348 - 28\times 0.000549)\\ &= 0.585\,362\text{ u}\end{aligned}$$

and so the binding energy is

$$\begin{aligned}E &= \delta c^2 = 0.585\,362 \times 931.5\text{c}^2 \quad \text{MeV c}^{-2}\\ &= 545.26 \quad \text{MeV}\end{aligned}$$

Hence the binding energy per nucleon is $\frac{E}{A} = \frac{545.26}{62} = 8.79 \quad \text{MeV}$.

This is the highest binding energy per nucleon.

2 The mass defect is

$$\begin{aligned}\delta &= 8m_p + 8m_n - (M_O - 8m_e)\\ &= 8\times 1.007\,276 + 8\times 1.008\,665 - (15.994 - 8\times 0.000549)\\ &= 0.137\,920\text{ u}\end{aligned}$$

and so the binding energy is

$$\begin{aligned}E &= \delta\text{c}^2 = 0.137\,920 \times 931.5\text{c}^2 \quad \text{MeV c}^{-2}\\ &= 128.47 \quad \text{MeV}\end{aligned}$$

Hence the binding energy per nucleon is $\frac{E}{A} = \frac{128.47}{16} = 8.03 \quad \text{MeV}$.

Consider now the reaction ${}^{16}_{8}\text{O} \to {}^{1}_{1}\text{p} + {}^{15}_{7}\text{N}$.

The mass difference is

$15.994 - 8 \times 0.000549 - (1.007\,276 + 15.000 - 7 \times 0.000549) = -0.012\,727\text{ u}$.

The negative sign implies that the reaction can take place only when energy is supplied to the oxygen nucleus.

This energy is $E = 0.012\,727 \times 931.5\text{c}^2 \quad \text{MeV c}^{-2} = 11.9 \quad \text{MeV}$.

3 The reaction is ${}^{1}_{0}\text{n} \to {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + \bar{\nu}$.

Neglecting the mass of the neutrino, the mass difference is

$1.008\,665 - 1.007\,276 - 0.000549 = 8.4 \times 10^{-4}\text{ u}$.

The energy released is then $E = 8.4 \times 10^{-4} \times 931.5 \quad \text{MeV c}^{-2} = 0.78 \quad \text{MeV}$.

4 **a** Using, $E = hf = \frac{hc}{\lambda}$ we find

$$\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3.0\times10^{8}}{0.051\times10^{6}\times1.6\times10^{-19}} = 2.44\times10^{-11}\ \text{m}.$$

b This is in the gamma ray area of the spectrum.

5 The mass difference is $234.043596-(230.037080+4.002603)=0.003913\ \text{u}$.
The negative sign implies that the reaction can take place only when energy is supplied to the oxygen nucleus.
This energy is $E = 0.003913\times931.5c^2\quad \text{MeV c}^{-2} = 3.65\quad \text{MeV}$.
(Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

6 **a** ${}^{236}_{92}\text{U} \rightarrow {}^{117}_{46}\text{Pd} + {}^{117}_{46}\text{Pd} + 2\,{}^{1}_{0}\text{n}$.

b Two neutrons are produced as well as photons.

c The mass difference is
$236.0455561-(2\times116.9178+2\times1.008665)=0.192626\ \text{u}$.
The energy is $E = 0.192626\times931.5\text{c}^2\quad \text{MeV c}^{-2} = 179\quad \text{MeV}$.
(Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

7 a The atomic numbers on the right add to $42+57=99$ so we need seven electrons to balance the sum.

b The mass numbers are already balanced ($236=95+139+2$). So the reaction is
${}^{235}_{92}\text{U}+{}^{1}_{0}\text{n} \rightarrow {}^{95}_{42}\text{Mo} + {}^{139}_{57}\text{La} + 2\,{}^{1}_{0}\text{n} + 7\,{}^{0}_{-1}\text{e}$.

We may ignore the seven electron masses if we use the atomic masses given.
So the mass difference is
$235.043992+1.008665-(94.905841+138.906342+2\times1.008665)=0.223144\ \text{u}$
and the energy released is $E = 0.223144\times931.5\text{c}^2\quad \text{MeV c}^{-2} = 208\quad \text{MeV}$.

8 The mass difference is
$235.043992+1.008665-(97.91276+134.9165+3\times1.008665)=0.197402\ \text{u}$.
The energy released is $E = 0.197402\times931.5\text{c}^2\quad \text{MeV c}^{-2} = 184\quad \text{MeV}$.

9 The mass difference is $2.014102+3.016049-(1.008665+4.002603)=0.018883\ \text{u}$.
This energy is $E = 0.018883\times931.5\text{c}^2\quad \text{MeV c}^{-2} = 17.6\quad \text{MeV}$.
(Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

10 The mass of the sun that will be converted into helium is $0.12\times 0.75\times 1.99\times 10^{30} = 1.79\times 10^{29}$ kg.

If there are N fusion reactions **per second,** the power of the sun will be given by $N\times 26.7\times 10^{6}\times 1.6\times 10^{-19} = 3.9\times 10^{26}$ kg and so $N = 9.13\times 10^{37}$.

The release of the 26.7 MeV of energy is associated with the conversion of four hydrogen nuclei into helium, i.e. a mass of hydrogen of $4\times 1.007\,276\times 1.661\times 10^{-27} = 6.69\times 10^{-27}$ kg.

So the mass converted per second is $9.13\times 10^{37}\times 6.69\times 10^{-27} = 6.1\times 10^{11}$ kg s^{-1}.

Hence, to convert the 1.79×10^{29} kg requires

$$\frac{1.79\times 10^{29}}{6.1\times 10^{11}}\text{ s} = 2.9\times 10^{17}\text{ s} \approx 9\times 10^{9}\text{ years}.$$

11 The mass difference is $1.007\,276 + (7.016 - 3\times 0.000549) - 2\times(4.002\,603 - 2\times 0.000549) = 0.018\,619$ u.

This corresponds to an energy $E = 0.018\,619\times 931.5c^2 \quad \text{MeV c}^{-2} = 17.3 \quad \text{MeV}$ not including the kinetic energy of the accelerated proton.

12 **a** High temperatures are required so that the nuclei will have sufficient kinetic energy in order to overcome their mutual electrostatic repulsion.

b High pressures are required so that the nuclei are sufficiently close to each other for a high probability of collision and hence fusion.

13 The formula for the mass defect given in the textbook is $\delta = Zm_{\text{p}} + (A-Z)m_{\text{n}} - M_{\text{nucleus}}$.

Now, $M_{\text{nucleus}} = M_{\text{atom}} - Zm_{\text{e}}$.

Hence,

$$\begin{aligned}\delta &= Zm_{\text{p}} + (A-Z)m_{\text{n}} - (M_{\text{atom}} - Zm_{\text{e}})\\ &= Z(m_{\text{p}} + m_{\text{e}}) + (A-Z)m_{\text{n}} - M_{\text{atom}}\\ &= ZM_{\text{H}} + (A-Z)m_{\text{n}} - M_{\text{atom}}\end{aligned}$$

where $M_{\text{H}} = m_{\text{p}} + m_{\text{e}}$ is the mass of the hydrogen atom.

14 **a** $Q_1 = \left(M_D + M_T - M_{He} - m_n\right)c^2$.

Now let us look at the binding energy of each nucleus involved in the reaction:

$$E_D = (m_p + m_n - M_D)c^2 \Rightarrow M_D c^2 = (m_p + m_n)c^2 - E_D$$
$$E_T = (m_p + 2m_n - M_T)c^2 \Rightarrow M_T c^2 = (m_p + 2m_n)c^2 - E_T$$
$$E_{He} = (2m_p + 2m_n - M_{He})c^2 \Rightarrow M_{He} c^2 = (2m_p + 2m_n)c^2 - E_{He}$$

Hence, replacing the masses in the equation for Q_1,

$$\begin{aligned} Q_1 &= \left(M_D + M_T - M_{He} - m_n\right)c^2 \\ &= \left((m_p + m_n - E_D) + (m_p + 2m_n - E_T) - (2m_p + 2m_n - E_{He}) - m_n\right)c^2 \\ &= E_{He} - (E_D + E_T) \end{aligned}$$

b $Q_2 = \left(M_U + M_{Zr} - M_{Te} - 2m_n\right)c^2$.

Working as in **a:**

$$E_U = (92m_p + 143m_n - M_U)c^2 \Rightarrow M_U c^2 = (92m_p + 143m_n)c^2 - E_U$$
$$E_{Zr} = (40m_p + 58m_n - M_{Zr})c^2 \Rightarrow M_{Zr} c^2 = (40m_p + 58m_n)c^2 - E_{Zr}$$
$$E_{Te} = (52m_p + 83m_n - M_{Te})c^2 \Rightarrow M_{Te} c^2 = (52m_p + 83m_n)c^2 - E_{Te}$$

$$\begin{aligned} Q_2 &= \left(M_U - M_{Zr} - M_{Te} - 2m_n\right)c^2 \\ &= \left((92m_p + 143m_n - E_U) - (40m_p + 58m_n - E_{Zr}) - (52m_p + 83m_n - E_{Te}) - 2m_n\right)c^2 \\ &= E_{Zr} + E_{Te} - E_U \end{aligned}$$

c The results in **a** and **b** show that, in general, the energy released can be found from the difference of the total binding energy **after** the reaction minus that **before** the reaction. Thus, to have energy released, the binding energy after the reaction must be greater than that before. The peak of the binding energy curve is at nickel. Elements to the right and left of nickel have lower binding energy per nucleon. The issue here is how to use the binding energy curve to show that energy will be released for fission (involving elements heavier than nickel) and fusion (involving elements lighter than nickel). Notice that we cannot prove mathematically that this is the case without knowing the mathematical equation of the binding energy curve. However, the fact that the curve rises for light elements up to nickel and then drops for elements heavier than nickel is indicative that energy is released in both fusion and fission reactions.

