Answers to Coursebook questions – Chapter 4.5

1 An observer on the approaching car will measure a higher frequency (f_1) than that emitted (f_0) because we have a case of the Doppler effect with an approaching source. The wave will then be reflected with frequency f_1. The car is now acting as an approaching source. The frequency received back at the source (f_2) will be higher than that emitted from the car. This is the case of a double Doppler effect.

2 From $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$ we deduce a speed of $v = c\frac{\Delta\lambda}{\lambda_0} = \frac{3\times10^8\times 3}{657} = 1.4\times10^6\ \text{m s}^{-1}$. The wavelength received is shorter than that emitted, meaning that the galaxy is approaching the Milky Way.

3 See **Figure 5.2** (page 245 in *Physics for the IB Diploma*).

4 This is a case of a source moving towards the observer, and so
$$f = f_0\frac{c}{c-v} = 500\frac{343}{343-40} = 566\ \text{Hz}$$

5 This is a case of a source moving away from the observer, and so
$$f = f_0\frac{c}{c+v} = 480\frac{343}{343+32} = 439\ \text{Hz}$$

6 This is a case of an observer approaching a stationary source and so the relevant formula is
$$f = f_0(1-\frac{v}{c}) = 512(1-\frac{12}{343}) = 494\ \text{Hz}$$

7 This is a case of an observer moving away from a stationary source and so the relevant formula is $f = f_0(1+\frac{v}{c}) = 628(1-\frac{25}{343}) = 674\ \text{Hz}$

8 The frequency received by the receding observer is (observer moving away) $f = f_0(1-\frac{v}{c})$.

The wave is reflected backwards. The moving observer now acts as the source of the waves, and the frequency emitted by this 'source' is $f = f_0(1-\frac{v}{c})$. The original source now acts as a stationary observer and so the frequency it receives is now $f\frac{1}{1+\frac{v}{c}} = f_0\frac{1-\frac{v}{c}}{1+\frac{v}{c}}$.

Hence $480 = 500\frac{1-\frac{v}{c}}{1+\frac{v}{c}}$.

Since $\frac{480}{500} = 0.96$ we have

$$0.96 = \frac{1-\frac{v}{c}}{1+\frac{v}{c}}$$

$$0.96 + 0.96\frac{v}{c} = 1 - \frac{v}{c}$$

$$1.96\frac{v}{c} = 0.04$$

$$\frac{v}{c} = 0.0204$$

$$v = 0.0204 \times 343 = 7.0 \text{ m s}^{-1}$$

Hint: You can put the equation $480 = 500\frac{1-\frac{v}{c}}{1+\frac{v}{c}}$ directly into the solver of your graphics calculator (with $x = \frac{v}{c}$) and get the answer immediately without any of the tedious algebra above.

9 This problem is similar to Q8. The frequency received by the stationary observer is (source moving towards) $f = \frac{f_0}{1-\frac{v}{c}}$. The wave is reflected backwards. The stationary observer now acts as the source of the waves, and the frequency emitted by this 'source' is $f = \frac{f_0}{1-\frac{v}{c}}$.

The original source now acts as a moving observer and so the frequency it receives is now $f(1+\frac{v}{c}) = f_0\frac{1+\frac{v}{c}}{1-\frac{v}{c}}$. Hence $512 = 500\frac{1+\frac{v}{c}}{1-\frac{v}{c}}$. Since $\frac{512}{500} = 1.024$ we have

$$1.024 = \frac{1+\frac{v}{c}}{1-\frac{v}{c}}$$

$$1.024 - 1.024\frac{v}{c} = 1 + \frac{v}{c}$$

$$2.024\frac{v}{c} = 0.024$$

$$\frac{v}{c} = 0.01186$$

$$v = 0.01186 \times 343 = 4.1 \text{ m s}^{-1}.$$

10 This is a case of a source moving towards and then away from the observer, and so

$f = f_0 \dfrac{c}{c-v} = 500\left(\dfrac{1}{1-\dfrac{7.5}{343}}\right) = 511\text{ Hz}$ when source approaches and

$f = f_0 \dfrac{c}{c+v} = 500\left(\dfrac{1}{1+\dfrac{7.5}{343}}\right) = 489\text{ Hz}$ when source moves away,

11 Assume, as this problem does, that the source and the observer move towards each other. As far as the observer is concerned the velocity of the source is $v_S + v_0$ and the speed of the wave is $v_0 + c$. So using the formula of the stationary observer and an approaching source, we have

$$f_0 = \frac{f_S}{1-\dfrac{v_S+v_0}{c+v_0}} = \frac{f_S}{\dfrac{c+v_0}{c+v_0} - \dfrac{v_S+v_0}{c+v_0}} = f_S \frac{c+v_0}{c-v_S}.$$

12 From the diagram in **Figure 5.2** (see page 245 in *Physics for the IB Diploma*) we see that for an observer for whom the source moves away the distance between successive wavefronts is larger than that measured by an observer at the source. This means that the wavelength measured will be larger. The frequency measured by the observer will be $f = \dfrac{f_0}{1+\dfrac{v}{c}}$ and so the wavelength measured will be $\lambda_0 = \dfrac{c}{f} = \dfrac{c}{\dfrac{f_0}{1+\dfrac{v}{c}}} = \dfrac{c}{f_0}(1+\dfrac{v}{c}) = \lambda_S(1+\dfrac{v}{c})$.

Hence the shift in wavelength is $\Delta\lambda = \lambda_0 - \lambda_S = \lambda_S(1+\dfrac{v}{c}) - \lambda_S = \lambda_S \dfrac{v}{c}$. Hence $\dfrac{\Delta\lambda}{\lambda_S} = \dfrac{v}{c}$.

13 **a** The frequency emitted is *f*. The observer is moving away so he receives a frequency $f_R = f\dfrac{c-v}{c}$. This frequency is reflected from the object, which now acts as a receding source. The frequency received back at the original source is then

$$f' = f_R \frac{c}{c+v} = \left(f\frac{c-v}{c}\right)\frac{c}{c+v} = f\frac{c-v}{c+v} = f\frac{1-\dfrac{v}{c}}{1+\dfrac{v}{c}}.$$

b If $\dfrac{v}{c}$ is small, then $f' \approx f(1-\dfrac{v}{c})(1+\dfrac{v}{c}) \approx f(1-2\dfrac{v}{c})$. Hence $\dfrac{\Delta f}{f} = \dfrac{2v}{c}$.

c **i** $\dfrac{\Delta f}{f} = \dfrac{2v}{c} \Rightarrow v = \dfrac{c\Delta f}{2f} = \dfrac{1500\times 2.4\times 10^3}{2\times 5.00\times 10^6} = 0.36\text{ m s}^{-1}$.

ii Because there is a range of speeds for the blood cells and the ultrasound is not incident normally on the cells.

14 The speed of a point on the sun's equator is

$$v = \frac{2\pi R}{T} = \frac{2\pi \times 7.00 \times 10^8}{27 \times 24 \times 60 \times 60} = 1.89 \times 10^3 \text{ m s}^{-1}.$$

The emitted frequency is $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{5.00 \times 10^{-7}} = 6.00 \times 10^{14}$ Hz .

The shifts are then $\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow \Delta f = f\frac{v}{c} = \frac{6.00 \times 10^{14} \times 1.89 \times 10^3}{3.00 \times 10^8} = 3.78 \times 10^9$ Hz .

15 **a** In this case the observer will first hear a sound of somewhat larger value than 500 Hz as the source approaches and then a sound of a single frequency lower than 500 Hz as the source moves away. There will be an abrupt change in frequency as the source moves past the observer.

b As in **a,** the frequencies between 500 Hz and 1000 Hz will be shifted to higher values as the source approaches and to lower values as the source moves away.

c As the source approaches, frequencies near 20 kHz move to the inaudible range and other lower frequencies take their place, so the highest frequency heard is still 20 kHz. The lowest heard is above 20 Hz. As the source moves away, the highest frequency heard will be less than 20 kHz and the lowest will be 20 Hz.

16 **a** There is no shift, since the velocity is at right angles to the direction of observation. The stars are neither approaching nor moving away from the observer at that time.

b The frequencies are

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{6.50 \times 10^{-7}} = 4.62 \times 10^{14} \text{ Hz} \quad \text{and} \quad f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{6.76 \times 10^{-7}} = 4.44 \times 10^{14} \text{ Hz}.$$

The emitted frequency is $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{6.58 \times 10^{-7}} = 4.56 \times 10^{14}$ Hz .

Therefore the speeds of the stars are

$$\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow v = \frac{c\Delta f}{f} = \frac{3.00 \times 10^8 \times 0.060 \times 10^{14}}{4.56 \times 10^{14}} = 3.95 \times 10^6 \text{ m s}^{-1} \text{ and}$$

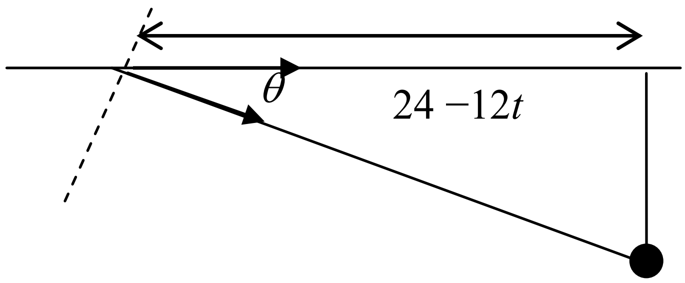
$$\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow v = \frac{c\Delta f}{f} = \frac{3.00 \times 10^8 \times 0.120 \times 10^{14}}{4.56 \times 10^{14}} = 7.90 \times 10^6 \text{ m s}^{-1}.$$

17

This is a rather difficult problem and is for students taking Math HL only.

a The sound increases and then decreases in intensity, giving a graph with the general shape as that in the answers (see page 802 in *Physics for the IB Diploma*).

b What counts is the component of velocity along the line of sight between the source and the observer. This component is $v\cos\theta = 12\cos\theta$.



If the initial position of the source is 24 m away and the observer is 4.0 m from the line of motion of the source, then as the source approaches:

$$f = \frac{850}{1 - \frac{12\cos\theta}{c}}$$

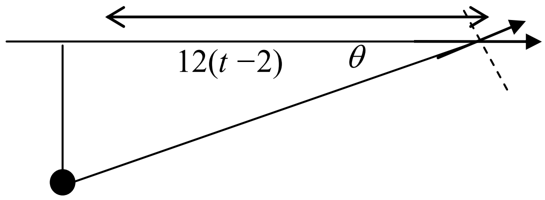
Now, we have that $\cos\theta = \frac{24 - 12t}{\sqrt{4.0^2 + (24 - 12t)^2}}$

and so

$$f = \frac{850}{1 - \frac{\frac{12(24-12t)}{\sqrt{4.0^2 + (24-12t)^2}}}{c}}$$

is what must be plotted from $t = 0$ to $t = 2.0\,\text{s}$.

At $t = 2.0\,\text{s}$ the source now moves away and then the appropriate diagram is:



And so $f = \dfrac{850}{1 + \dfrac{12\cos\theta}{c}}$.

Now, we have that $\cos\theta = \dfrac{12(t-2)}{\sqrt{4.0^2 + (12(t-2))^2}}$ and so

$$f = \frac{850}{1 + \dfrac{\dfrac{12(12(t-2))}{\sqrt{4.0^2 + (12(t-2))^2}}}{c}}$$

which must be plotted for $t > 2.0\,\text{s}$ to give the graph in the answers (see page 802 in *Physics for the IB Diploma*).

c With acceleration, the formulas for frequency change to:

$$f = \frac{850}{1 - \dfrac{\dfrac{12(24 - 10t - t^2)}{\sqrt{4.0^2 + (24 - 10t - t^2)^2}}}{c}}$$

for the source approaching, and

$$f = \frac{850}{1 + \dfrac{\dfrac{12(10(t-2) + (t-2)^2)}{\sqrt{4.0^2 + (10(t-2) + (t-2)^2)^2}}}{c}}$$

for the source moving away (since the source will stay take 2 s to get past the observer).

The point of the exercise with acceleration is this: with constant acceleration, as the source approaches, the angle θ gets larger, $v\cos\theta = 12\cos\theta$ gets smaller and so, since $f = \dfrac{850}{1 - \dfrac{12\cos\theta}{c}}$, the frequency gets smaller gradually, as shown in the graph.

With acceleration, we have two competing effects as the source approaches: a decreasing frequency due to increasing angle, as in the case of constant velocity, but now an increasing frequency as well due to the increased speed. At some point the increased speed effect dominates, which is why a maximum in the observed frequency right before the source reaches the observer as shown in the graph.

18 **a** The observer is moving towards the source, and so $f = f_0(1+\frac{v}{c})$,

i.e. $580 = 530(1+\frac{v}{c}) \Rightarrow v = 32\ \mathrm{m\,s^{-1}}$.

b **i** $\lambda_S = \frac{c}{f_0} = \frac{340}{530} = 0.64\ \mathrm{m}$.

ii $\lambda_{obs} = \frac{c+v}{f} = \frac{340+32}{580} = 0.64\ \mathrm{m}$, i.e. the same as at the source.

19 **a**

$$\Delta f = f\frac{v}{c} \Rightarrow f' = f \pm f\frac{v}{c} = f(1\pm\frac{v}{c}).$$

Hence $\lambda' = \frac{c}{f'} = \frac{c}{f(1\pm\frac{v}{c})} \approx \lambda(1\,\mathrm{m}\frac{v}{c})$.

The shift in wavelength is thus

$$\Delta\lambda = |\lambda' - \lambda| = \frac{\lambda v}{c}$$

b The wavelength shift is $\Delta\lambda = 5.65\times10^{-7} - 5.48\times10^{-7} = 0.170\times10^{-7}\ \mathrm{m}$ and so from

$$\Delta\lambda = \frac{\lambda v}{c},\quad v = \frac{c\Delta\lambda}{\lambda} = \frac{3.00\times10^{8}\times0.170\times10^{-7}}{5.48\times10^{-7}} = 9.3\times10^{7}\ \mathrm{m\,s^{-1}}$$