Answers to Coursebook questions – Chapter 4.1

1 An oscillation is any motion in which the displacement of a particle from a fixed point keeps changing direction and there is a periodicity in the motion, i.e. the motion repeats in some way. In simple harmonic motion, the displacement from an equilibrium position and the acceleration are proportional to and opposite to each other.

2 In an SHM oscillation the period is independent of the amplitude and the acceleration is opposite and proportional to the displacement from equilibrium.

3 It is an oscillation, since we may define the displacement of the particle from the middle point and in that case the displacement changes direction and the motion repeats. The motion is not simple harmonic, however, since there is no acceleration that is proportional (and opposite) to the displacement.

4 It is an oscillation, since the motion repeats. The motion is not simple harmonic, however, since the acceleration is constant and is not proportional (and opposite) to the displacement.

5 We know that a mass at the end of a spring when displaced will perform simple harmonic oscillations with a period given by $T = 2\pi\sqrt{\frac{m}{k}}$. Hence, measuring the spring constant and the period of oscillations allows determination of the mass.

6 They are not simple harmonic because, as shown (see page 201 in *Physics for the IB Diploma*), the restoring force, although opposite to, is not proportional to the displacement away from the equilibrium position. If, however, the amplitude of oscillations is small, the force does become approximately proportional to the displacement and the oscillations are then approximately simple harmonic.

7 At $t = 0$ the equation says that $x = A\cos\phi$.
The next time x assumes this value is at a time given by $A\cos(\omega T + \phi) = A\cos\phi$.
Thus we must solve the equation $\cos(\omega T + \phi) = \cos\phi$. This means that the angles $\omega T + \phi$ and ϕ differ by 2π and so solutions are $\omega T + \phi = \phi + 2\pi \Rightarrow T = \frac{2\pi}{\omega}$.

8 **a** At $t = 0$ we have $y = 5.0\cos(0) = 5.0\text{ mm}$.

b At $t = 1.2\text{ s}$ we use the calculator (in **radian** mode) to find $y = 5.0\cos(2\times 1.2) = -3.7\text{ mm}$.

c $-2.0 = 5.0\cos(2t) \Rightarrow 2t = \cos^{-1}(-\frac{2}{5}) = 1.98 \Rightarrow t = 0.99\text{ s}$.

d Use $v = \pm\omega\sqrt{A^2 - x^2}$.
We know that $\omega = 2.0\text{ s}^{-1}$.
Therefore, $6.00 = \pm 2.0\sqrt{25 - x^2} \Rightarrow 25 - x^2 = 9.0 \Rightarrow x = \pm 4.00\text{ mm}$.

9 **a** The equation is simply $y = 8.0\cos(2\pi \times 14t) = 8.0\cos(28\pi t)$.

b The velocity is therefore $v = -8.0 \times 28\pi \sin(28\pi t)$ and the acceleration is $a = -8.0 \times (28\pi)^2 \cos(28\pi t)$.
At $t = 0.025\text{ s}$ we evaluate (in radian mode):
$y = 8.0\cos(28\pi \times 0.025) = -4.7\text{ cm}$,
$v = -8.0 \times 28\pi \sin(28\pi \times 0.025) = -5.7\text{ m s}^{-1}$ and
$a = -8.0 \times (28\pi)^2 \cos(28\pi \times 0.025) = 3.6 \times 10^2\text{ m s}^{-2}$.

10 The angular frequency is $\omega = 2\pi f = 2\pi \times 460 = 920\pi$.
The maximum velocity is $\omega A = 920\pi \times 5.0 \times 10^{-3} = 14\text{ m s}^{-1}$ and
the maximum acceleration is $\omega^2 A = (920\pi)^2 \times 5.0 \times 10^{-3} = 4.2 \times 10^4\text{ m s}^{-2}$.

11 **a** The equation of the string may be rewritten as $y = \left(6.0\sin(\pi x)\right)\cos(2\pi \times 520t)$, from which we deduce that the frequency of all points is 520 Hz and that the phase of all points is zero.

b From **a** the amplitude is $A = 6.0\sin(\pi x)$ and so is different for different points on the string.

c The maximum amplitude is obtained when $\sin(\pi x) = 1$, i.e. the maximum amplitude is 6.0 mm.

d The displacement is always zero at the ends of the string, in particular at the right end, where $x = L$, the length of the string. The displacement is zero **all the time** when $6.0\sin(\pi x) = 0$, i.e. when $\pi x = \pi \Rightarrow x = 1.0\text{ m}$.

e When $x = \frac{3L}{4} = 0.75\text{ m}$ the amplitude is $6.0\sin(\pi x) = 6.0\sin(0.75\pi) = 4.2\text{ mm}$.

12 At A the acceleration is maximum in magnitude and is directed to the right.
At B it is directed to the right and has smaller magnitude than at A.
At C we have the equilibrium position and so the acceleration is zero.
At D the acceleration is to the left and has the least magnitude compared to A and B.
Net force: since $F = ma$ whatever is true for acceleration is also true for net force.

13 **a** At maximum displacement we will have the maximum acceleration, i.e. $a_{max} = \omega^2 A$. The amplitude is $A = 4.5\text{ cm}$ and $\omega = 2\pi f = 2\pi \frac{4500}{60}\text{ s}^{-1}$. Therefore,

$$a_{max} = \left(2\pi \frac{4500}{60}\right)^2 \times 4.5 \times 10^{-2} = 9993 \approx 1.0 \times 10^4\text{ m s}^{-2}.$$

b As the piston moves past its equilibrium position it has the maximum speed and so $v_{max} = 2\pi \frac{4500}{60} \times 4.5 \times 10^{-2} = 21\text{ m s}^{-1}$.

c $F_{max} = ma_{max} = 0.25 \times 9993 \approx 2.5 \times 10^3\text{ N}$.

14 **a** The area is approximately 0.50 cm (the exact value is 0.51 cm) (ideally the graph should have been given on graph paper and we would then count squares).

b This is the displacement from when the velocity is zero to when it is zero again, i.e. from one extreme position until the other, i.e. twice the amplitude.

c The period is 0.4 s and so the equation for displacement is

$x = -0.25\sin(\frac{2\pi t}{0.4}) = -0.25\sin(5\pi t)$.

15 We need to graph the equation $a = -\omega^2 x$, where $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12.57\ \text{s}^{-1}$.

The slope would be $\omega^2 = 158\ \text{s}^{-2}$ or just 1.58 since we are plotting cm on the horizontal axis.
This gives the graph in the answers (see page 799 in *Physics for the IB Diploma*).

16 **a** The defining relation for SHM is that $a = -\omega^2 x$ which implies that a graph of acceleration versus displacement is a straight line through the origin with negative slope just as the given graph.

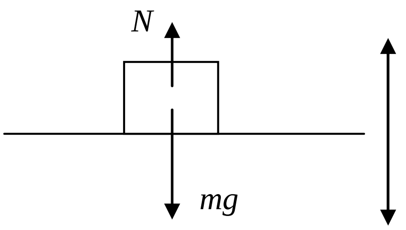
b The slope of the graph gives $-\omega^2$.
The measured slope is $\frac{1.5}{0.10} = -15\ \text{s}^{-2}$ and so $\omega = \sqrt{15} = 3.873\ \text{s}^{-1}$.
Thus the period is $T = \frac{2\pi}{3.873} = 1.6\ \text{s}$.

c The maximum velocity is $\omega A = 3.873 \times 0.10 = 0.39\ \text{m s}^{-1}$.

d The maximum net force is $ma = m\omega^2 A = 0.150 \times 3.873^2 \times 0.10 = 0.23\ \text{N}$.

e The total energy is $E_T = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}0.150 \times 3.873^2 \times 0.10^2 = 0.012\ \text{J}$.

17 **a** The forces on the mass when the plate is at the top are shown below:



The net force is $mg - N = ma$. Since we have simple harmonic motion $a = \omega^2 x = 4\pi^2 f^2 x$ in magnitude, and the largest acceleration is obtained when $x = A$, the amplitude of the oscillation. The frequency is 5.0 Hz. The critical point is when $N = 0$, i.e. $g = 4\pi^2 f^2 A$ and so $A = \dfrac{g}{4\pi^2 f^2} = \dfrac{9.8}{4\pi^2 \times 25} = 0.0099\ \text{m}$.

The amplitude must not exceed this value.

b At the lowest point:

$$N - mg = ma = m4\pi^2 f^2 A$$
$$\Rightarrow N = mg + m4\pi^2 f^2 A$$
$$N = 0.120 \times 9.8 + 0.120 \times 4 \times \pi^2 \times 25 \times 0.0099$$
$$N = 2.35\ \text{N}$$

18 **a** We have simple harmonic oscillations about a mean value of scale reading of 70 kg which is the mass of the passenger. The period is 10 s.

b At the top point of the oscillations the reading is 50 kg and at the bottom it is 90 kg. At the lowest point (see **Q17**) we have that

$$N = mg + m4\pi^2 f^2 A$$
$$= mg + \frac{m4\pi^2 A}{T^2}$$
$$90 \times 9.8 = 70 \times 9.8 + \frac{70 \times 4 \times \pi^2 \times A}{10^2}$$
$$A = 7.1\ \text{m}$$

(This passenger has probably regretted buying a ticket on this particular cruise.)

19 **a** The volume within the sphere of radius x is $\frac{4\pi x^3}{3}$ and that of the entire sphere is $\frac{4\pi R^3}{3}$; therefore the mass enclosed is the fraction $M\frac{x^3}{R^3}$.

b $F = G\frac{\frac{Mx^3}{R^3}m}{x^2} = \frac{GMmx}{R^3}$.

c The acceleration of the mass is given by $ma = -\frac{GMmx}{R^3} \Rightarrow a = -\frac{GM}{R^3}x$, which is the condition for SHM with $\omega^2 = \frac{GM}{R^3}$.

d $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}$.

e $T = 2\pi\sqrt{\frac{(6.4\times10^6)^3}{6.67\times10^{-11}\times6.0\times10^{24}}} = 5085\text{ s} = 85\text{ min}$.

f From gravitation we know that

$\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2 \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$ as in **d**.

20 **a** The tension in each string is T. Taking components of the two tension forces we see that the horizontal components $T\cos\theta$ cancel out, leaving a downward net from the vertical components of $2T\sin\theta$.

b From $F_{net} = ma$ we get $ma = -2T\sin\theta - mg$ and hence $a = \frac{-2T\sin\theta}{m} - g$.

Since the tension is very large and the mass is small, the first term is bound to dominate, and so we may neglect the acceleration of gravity in the expression above. Hence, approximately, $a = -\frac{2T\sin\theta}{m}$.

Now $\sin\theta \approx \tan\theta = \frac{x}{\frac{L}{2}} = \frac{2x}{L}$ and finally $a = -\frac{4T}{mL}x$.

This means we will have simple harmonic oscillations with $\omega^2 = \frac{4T}{mL}$.

c The period is given by $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mL}{4T}}$.

21 The period of oscillation does not depend on the acceleration of gravity. Accelerating may be thought of as changing the acceleration of gravity (increasing it for upwards accelerations and decreasing it for downward accelerations). Hence the period will be unaffected.

22 The displacement is in general $y = A\cos(\omega t + \phi)$ and so the velocity is $v = -\omega A\sin(\omega t + \phi)$.

So the initial displacement is $x_0 = A\cos\phi$ and the initial velocity is $v_0 = -\omega A\sin\phi$. In other words,

$$\cos\phi = \frac{x_0}{A}$$

$$\sin\phi = -\frac{v_0}{\omega A}.$$

But $\cos^2\phi + \sin^2\phi = 1$, i.e. $\left(\frac{x_0}{A}\right)^2 + \left(\frac{v_0}{\omega A}\right)^2 = 1 \Rightarrow A = \sqrt{x_0^{\,2} + \frac{v_0^{\,2}}{\omega^2}}$.

23 **a** In this case the putty will stick to the mass because of frictional forces between them, and so some energy will be lost reducing the amplitude of oscillation. The mass will increase, increasing the period of oscillations.

b When the block is momentarily at rest the putty will rest on it without any frictional forces acting and so the energy of the spring–mass system will be unaffected. Hence the amplitude of oscillation will also be unaffected. Since the mass has increased, however, the period will increase as well.

24 **a** The amplitude is 0.360 m. The frequency is $2\pi f = 6.80 \Rightarrow f = 1.0823 \approx 1.08\text{ Hz}$ and the period is $T = \frac{1}{f} = \frac{1}{1.0823} = 0.924\text{ s}$.

b The total energy is $E_T = \frac{1}{2}\omega^2 A^2$. Now $\omega = 2\pi f = 6.80\text{ s}^{-1}$ and so

$$E_T = \frac{1}{2} \times 1.80 \times (6.80)^2 \times (0.360)^2 = 5.40\text{ J}.$$

c The elastic potential energy is

$$E_P = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2} \times 1.80 \times (6.80)^2 \times (0.125)^2 = 0.650\text{ J}.$$

The kinetic energy is therefore $E_K = 5.40 - 0.650 = 4.75\text{ J}$.

25 **a** When extended by an amount *x*, the force pulling back on the body is $2kx$ and so $ma = -2kx \Rightarrow a = -\frac{2k}{m}x$, and so $\omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \times 120}{2.0}} = 10.95\text{ s}^{-1}$, giving a period of $T = \frac{2\pi}{\omega} = \frac{2\pi}{10.95} = 0.57\text{ s}$.

b With the springs connected this way, and the mass pulled to the side by a small amount, one spring will be compressed and the other extended. Hence the net force on the mass will still be $2kx$ so the period will not change.

26 **a** At the top the woman's total energy is gravitational potential energy equal to mgh, where h is the height measured from the lowest position that we seek.
At the lowest position all the gravitational potential energy has been converted into elastic energy $\frac{1}{2}kx^2$ and so $mgh=\frac{1}{2}kx^2$.

Since $h=15+x$ we have that $mgh=\frac{1}{2}k(h-15)^2$.

We must now solve for the height h:

$$60\times 10\times h=\frac{1}{2}\times 220\times (h-15)^2$$
$$600h=110(h^2-30h+225)$$
$$110h^2-3900h+24750=0$$
$$11h^2-390h+2475=0$$

The physically meaningful solution is $h=27.2\text{ m}$.

b The forces on the woman at the position in **a** are her weight vertically downwards and the tension in the spring upwards. Hence the net force is
$F_{net}=T-mg=kx-mg=220\times(27.2-15)-600=2084\text{ N}$; hence
$a=\frac{F_{net}}{m}=\frac{2084}{60}=34.7\text{ m s}^{-2}$

c Let x be the extension of the spring at some arbitrary position of the woman. Then the net force on her is $F_{net}=T-mg=kx-mg$ directed upwards, i.e. opposite to the direction of x. So $ma=-(kx-mg)$. The acceleration is not proportional to the displacement, so it looks as if we do not have SHM. But we must measure displacement from an equilibrium position. This is when the extension of the spring is x_0 and $kx_0=mg$. In other words call the displacement to be $y=x-x_0$. Then $ma=-(k(y+x_0)-mg)=-ky-kx_0+mg=-ky$ since $kx_0=mg$. Hence, we do have the condition for SHM. And so $a=-\frac{k}{m}y$ so that $\omega^2=\frac{k}{m}\Rightarrow \omega=1.91\text{ s}^{-1}$ and finally $T=\frac{2\pi}{\omega}=3.28\text{ s}$.

d She will come to rest when the tension in the spring equals her weight, i.e. when $kx_0=mg\Rightarrow x_0=\frac{mg}{k}=\frac{60\times 10}{220}=2.7\text{ m}$. Hence the distance from the top is $15+2.7=17.7\text{ m}$.

e Since she will perform many oscillations before coming to rest, the oscillations are underdamped.

f It has been converted to other forms of energy, mainly thermal energy in the air and at the point of support of the spring.

27 An SHM oscillation is characterised by the equation $a = -\omega^2 x$. With damping there will be, in addition, a force opposing the motion, which is usually proportional to the speed of the particle. Depending on the magnitude of this force we can have three distinct cases:

i A very small opposing force in which case the oscillations die out slowly.

ii A specific larger force which makes the particle return to its equilibrium position as fast as possible but without oscillations.

iii Finally, any force larger than the force in **ii,** in which case the particle again returns to its equilibrium position without oscillations but in a time longer than that in **ii**.

28 Desirable:

i Shock absorbers in cars. It is desirable because without damping every time the car hits a bump the car (and its passengers) would suffer uncomfortable oscillations up and down.

ii Scales that are used in weighing things. If the oscillations of the pointer were not damped, the pointer would oscillate for a long time before settling down to the true reading of the mass of the object being weighed.

Undesirable:

i The oscillations in your grandfather's clock hanging on the wall.

ii The oscillations of a child in a swing when the father is too lazy to push!

29 **a** The ratio of successive amplitudes does appear to be constant at about $\frac{A_{n+1}}{A_n} \approx 0.6$.

b The fraction of energy lost is $\frac{A^2{}_n - A^2{}_{n+1}}{A^2{}_n} \approx 1 - 0.6^2 = 0.64 \approx 0.6$.

c The ratio $\frac{A_{n+1}}{A_n}$ of successive amplitudes would now be smaller than 0.6 and the period would get very slightly longer.

30 See **Figures 1.17, 1.18** and **1.19** (pages 207 and 208 in *Physics for the IB Diploma*).

31 They should ideally be critically damped so that the passengers do not go on oscillating for a long time after the car hits a bump in the road as they would in the case of underdamped oscillations. They should not be overdamped since in that case the car would return to its normal (and safe) condition for driving in a longer time.

32 As hinted at in **Q31**, overdamped shock absorbers might make the passenger feel the bump less but it would take a longer time to bring the car into its normal driving condition. This is not safe as the brakes will take a longer distance to bring the car to a stop in this case.

33 See **Figures 1.17, 1.18** and **1.19** (pages 207 and 208 in *Physics for the IB Diploma*).

34 **a** The angular frequency is $\omega = \sqrt{\frac{k}{m}}$ and the frequency is found from $\omega = 2\pi f$.

b The displacement is in general $y = A\cos(\omega t + \phi)$ and so the velocity is $v = -\omega A\sin(\omega t + \phi)$. Hence

$$\cos(\omega t + \phi) = \frac{x}{A}$$

$$\sin(\omega t + \phi) = -\frac{v}{\omega A}$$

But $\cos^2(\omega t + \phi) + \sin^2((\omega t + \phi) = 1$, i.e. $\left(\frac{x}{A}\right)^2 + \left(\frac{v}{\omega A}\right)^2 = 1$.

c The graph is an ellipse centred at the origin.

d The area of the ellipse in **c** is $\pi A^2\omega$.

e With damping, the energy of the system will be reduced and the amplitude *A* will decrease. Hence the area of the ellipse will decrease. This will happen as the graph in **c** becomes an elliptical spiral that closes onto the origin.

35 Free oscillations are oscillations in which no other force, other than the restoring force that is opposite and proportional to displacement, act on the system. In the case of forced oscillations there are other additional forces acting on the system.

36 Any structure has its own natural frequency of oscillation. When the structure is subjected to an external periodic force the amplitude of the resulting oscillations will be the largest when the natural frequency of the system and that frequency of the external force are the same. This is called a state of resonance.

An example of resonance is the microwave oven, in which microwaves of frequency equal to that of the vibrating water molecules are directed at food. The water molecules oscillate with the greatest amplitude in this case, which means that the temperature of the water molecules in the food rises and the food is warmed.

37 In case the periodic marching gets the bridge in resonance with dangerous large-amplitude oscillations that might break the bridge.

38 **a** $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

b The net force on the mass is the tension in the spring, kx and so $m\frac{v^2}{L_0 + x} = kx$ where L_0 is the natural length of the spring.

Now, $v = \frac{2\pi R}{T} = 2\pi R f$ and so $m\frac{(2\pi R f)^2}{L_0 + x} = kx$.

We must solve this equation to find the extension x:

$$4m\pi^2(L_0 + x)f^2 = kx$$

$$4m\pi^2 L_0 f^2 = x(k - 4m\pi^2 f^2)$$

$$x = \frac{4m\pi^2 L_0 f^2}{k - 4m\pi^2 f^2}$$

c The denominator in **b** is $k - 4m\pi^2 f^2 = k - 4m\pi^2 \frac{1}{4\pi^2}\frac{k}{m} = 0$, which is impossible.

d We have a case of resonance where a displacement becomes uncontrollably large. This is an idealised situation, though, where damping has been completely neglected. In a real situation with friction and damping this would not happen.