Answers to Coursebook questions – Chapter 3.4

1 $W = +p\Delta V = 5.4\times10^{5}\times(4.3-3.6)\times10^{-3} = 378 \approx 380\ \text{J}$.

2 **a** $Q = \Delta U + W$ with $W = -6500\ \text{J}$ and $\Delta U = 0$. Hence $Q = -6500\ \text{J}$; the thermal energy is removed to the system.

b The adiabatic curve would be steeper and so the area under it would be greater. The work done would be numerically greater than $6500\ \text{J}$.

3 See **Figure 4.5 (c)** (see page 186 in *Physics for the IB Diploma*). The work done would be less for the adiabatic expansion.

4 See **Figure 4.5 (d)** (see page 186 in *Physics for the IB Diploma*). The work done would be more (numerically) for the adiabatic compression.

5 For gas X the change is isothermal; hence ΔU and ΔT must be zero. We have an expansion; hence the work is positive.
For gas Y, there is no change in volume, and hence no work done. From the gas laws the temperature and hence the internal energy increase.
For gas Z using the gas laws the temperature and hence the internal energy increase.

	W	ΔU	ΔT
Gas X	positive	zero	zero
Gas Y	zero	positive	positive
Gas Z	positive	positive	positive

6 **a** From the first law we have that $Q = \Delta U + W$. Since the compression is adiabatic, $Q = 0$ and $W < 0$ (compression). Hence $\Delta U = -W > 0$. Hence the temperature increases.

b An adiabatic compression involves a piston, for example, rapidly compressing a gas. This means that the molecules, on the average, will bounce back from the piston with a greater speed than before. Hence the average random kinetic energy of the molecules will go up and thus so will the temperature since temperature and average kinetic energy are proportional.

7 **a** $W = +p\Delta V = 6.0\times10^{6}\times0.40 = 2400\ \text{kJ}$

b $\frac{V_1}{T_1} = \frac{V_2}{T_2}$; hence $\frac{0.20}{300} = \frac{0.60}{T_2}$, giving $T_2 = 900\ \text{K}$

c $Q = \Delta U + W = 3600 + 2400 = 6000\ \text{kJ}$

8 Since $Q = \Delta U + W$, to have $\Delta U = 0$ means that $Q = W$, i.e. all the thermal energy supplied is used to perform work.

9 **a** We must first determine whether the gas is expanding or whether it is being compressed. Since $pV = nRT$, it follows that $p = \frac{nRT}{V}$.

For **constant volume**, the graph of pressure versus temperature would be a straight line through the origin. Another graph of pressure versus temperature for a gas at a **different and greater** constant volume would also be a straight line through the origin but with a smaller slope. Hence the volume is decreasing as we move from P to Q and so the work is being done on the gas.

b From $Q = \Delta U + W$ we see that $\Delta U < 0$ since the temperature is decreasing and, since the gas is being compressed, we also have that $W < 0$.
Hence $Q < 0$ and thermal energy is being removed from the gas.

10 For gas X, $W = 0$ and so $Q = \Delta U_X$. For gas Y, $Q = \Delta U_Y + W$.
Hence, $\Delta U_X = \Delta U_Y + W$, and so the gas at constant volume has the greater temperature increase.

11 For gas X, $W = 0$ and so $Q_X = \Delta U$. For gas Y, $Q_Y = \Delta U + W$.
Since the temperature change is the same in both cases the ΔU is the same. Hence, $Q_Y = Q_X + W$ and so the gas at constant pressure has to be given more thermal energy than the gas X.

12 From $Q = \Delta U + W$ we have that in the constant volume case, $W = 0$ and so $nc_V \Delta T = \Delta U + 0$.

In the constant pressure case, $W = p\Delta V$ and so $nc_P \Delta T = \Delta U + p\Delta V$.

Let the temperature change be the same in both cases. Then ΔU is the same in both cases.
Then, $nc_V \Delta T = nc_P \Delta T + p\Delta V$.

Now from $pV = nRT$ it follows that $p\Delta V = nR\Delta T$ and so $nc_V \Delta T = nc_P \Delta T + nR\Delta T$.

Simplifying gives $c_P - c_V = R$.

Working further, and using $U = \frac{3}{2}nRT$, it follows from $nc_V \Delta T = \Delta U$ that

$nc_V \Delta T = \frac{3}{2}nR\Delta T$, i.e. $c_V = \frac{3R}{2}$.

Hence, from $c_P - c_V = R$, it follows that $c_P = \frac{5R}{2}$.

13 **a** A and B have the same pressure and so $\frac{V_1}{T_1}=\frac{V_2}{T_2}$, i.e. $\frac{0.1}{800}=\frac{0.4}{T_2}$; hence $T_2=3200\,\text{K}$.

B and C have the same volume ; hence $\frac{P_1}{T_1}=\frac{P_2}{T_2}$, i.e. $\frac{4}{3200}=\frac{2}{T_2}$; hence $T_2=1600\,\text{K}$.

C and D have the same pressure and so $\frac{V_1}{T_1}=\frac{V_2}{T_2}$, i.e. $\frac{0.4}{1600}=\frac{0.1}{T_2}$; hence $T_2=400\,\text{K}$.

b From A to B: .$Q=\Delta U+W$. We are given that $\Delta U=180\ \text{kJ}$ and $W=+p\Delta V=4.0\times10^5\times0.3=120\ \text{kJ}$. Hence $Q=180+120=300\ \text{kJ}$.

From B to C: we are given that $\Delta U=-120\ \text{kJ}$ and since $W=0$ we get $Q=-120\,\text{kJ}$.

From C to D: we are given that $\Delta U=-90\ \text{kJ}$ and $W=-p\Delta V=2.0\times10^5\times0.3=-60\ \text{kJ}$. Hence $Q=-90-60=-150\ \text{kJ}$.

Finally, from D to A: $\Delta U=30\ \text{kJ}$ and $W=0$ so that $Q=+30\ \text{kJ}$.

14 $W=\int_{V_1}^{V_2} p\,dV$. Since $pV=nRT$ it follows that $p=\frac{nRT}{V}$ and so $W=\int_{V_1}^{V_2}\frac{nRT}{V}\,dV$.

Since nRT is constant, $W=nRT\int_{V_1}^{V_2}\frac{1}{V}\,dV=nRT\ln V\Big|_{V_1}^{V_2}=nRT(\ln V_2-\ln V_1)=nRT\ln\frac{V_2}{V_1}$.

15 **a** The water loses entropy and the ice gains entropy.

b During the melting of ice the ice gains in entropy and the water loses entropy.

c The colder water is gaining entropy and the rest of the water is losing entropy.

