Answers to Coursebook questions – Chapter 3.3

1 From $\frac{P_1V_1}{n_1T_1}=\frac{P_2V_2}{n_2T_2}$ we deduce that $\frac{V_1}{T_1}=\frac{V_2}{T_2}$, i.e. that $\frac{2.0}{293}=\frac{V_2}{353}$;
hence $V_2=2.41\,\text{L}$.

2 From $\frac{P_1V_1}{n_1T_1}=\frac{P_2V_2}{n_2T_2}$ we deduce that $\frac{P_1}{T_1}=\frac{P_2}{T_2}$, i.e. that $\frac{12.0}{295}=\frac{P_2}{393}$;
hence $P_2=16.0\,\text{atm}$.

3 From $\frac{P_1V_1}{n_1T_1}=\frac{P_2V_2}{n_2T_2}$ we deduce that $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$, i.e. $\frac{4.00\times V_1}{301}=\frac{3.00\times 0.45}{313}$;
hence $V_1=0.325\,\text{L}$.

4 Since the radius doubles, the volume increases by a factor of 8. Thus pressure decreases by a factor of 8.

5 A quantity of 12.0 kg of helium corresponds to $\frac{12\times10^3}{4}=3.0\times10^3$ mol. Then from the gas law, $pV=nRT$ we get $P=\frac{nRT}{V}=\frac{3.00\times10^3\times8.31\times293}{5.00\times10^{-3}}=1.46\times10^9\ \text{m}^3$.

6 A quantity of 12.0 kg of helium corresponds to $\frac{12\times10^3}{4}=3.0\times10^3$ mol.

From the gas law, $pV=nRT$ we get $n=\frac{PV}{RT}=\frac{4.00\times10^5\times12.0\times10^{-3}}{8.31\times293}=1.97$ mol.

Since the mass of one mole of carbon dioxide (CO_2) is 44 g, we need $44\times1.97=87\ g$.

7 We use $\frac{P_1V_1}{n_1T_1}=\frac{P_2V_2}{n_2T_2}$ to get $\frac{P}{n_1}=\frac{\frac{P}{2}}{n_2}$ and hence $n_2=\frac{n_1}{2}$.

In other words, to reduce the pressure to half its original value, half the molecules must leave the container. The original number of molecules can be found using $pV=nRT$ to get $n=\frac{PV}{RT}=\frac{5.00\times10^5\times300\times10^{-6}}{8.31\times300}=0.0602$,

and hence $N=0.0602\times6.02\times10^{23}=3.62\times10^{22}$.

So we will have to lose $\frac{N}{2}=1.81\times10^{22}$ molecules.

This will take $\frac{1.81\times10^{22}}{3.00\times10^{19}}=603\ \text{s}\approx10\ \text{min}$.

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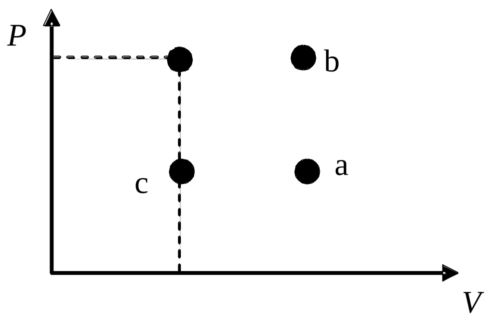

9 **a** At B use $\frac{V_1}{T_1}=\frac{V_2}{T_2}$ (compare A to B) to get $\frac{2}{300}=\frac{8}{T}$; hence $T=1200\text{ K}$.

At C use $\frac{P_1}{T_1}=\frac{P_2}{T_2}$ (compare B to C) to get $\frac{4}{1200}=\frac{2}{T}$; hence $T=600\text{ K}$.

At D use $\frac{V_1}{T_1}=\frac{V_2}{T_2}$ (compare C to D) to get $\frac{8}{600}=\frac{2}{T}$; hence $T=150\text{ K}$.

b The internal energy is greatest when the temperature is greatest, hence B.

10 Let there be n_1 moles of the gas in the left container and n_2 in the right. Then it must be true (using $n=\frac{PV}{RT}$) that $n_1=\frac{12\times10^5\times6.0\times10^{-3}}{RT}$ and $n_1=\frac{6.0\times10^5\times3.0\times10^{-3}}{RT}$.

When the gases mix we will have n_1+n_2 moles in a volume of 9.0 L and so

$$n_1+n_2=\frac{P\times9.0\times10^{-3}}{RT}.$$

Hence $\frac{12\times10^5\times6.0\times10^{-3}}{RT}+\frac{6.0\times10^5\times3.0\times10^{-3}}{RT}=\frac{P\times9.0\times10^{-3}}{RT}$.

This means that $P=\frac{12\times10^5\times6.0+6.0\times10^5\times3.0}{9.0}=10\times10^5\text{ Pa}=10\text{ atm}$.

11 **a** The cross-sectional area of the piston is $A=\frac{V}{h}=\frac{0.050}{0.500}=0.10\text{ m}^2$.

The pressure in the gas is constant and equal to $P=\frac{F}{A}=\frac{10.0\times10}{0.010}=1.0\times10^4\text{ Pa}$.

b From the gas law, $n=\frac{PV}{RT}=\frac{1.0\times10^4\times0.050}{8.31\times292}=0.206$.

The number of molecules is then $N=0.206\times6.02\times10^{23}=1.24\times10^{23}$.

c From $\frac{V_1}{T_1}=\frac{V_2}{T_2}$ we get $\frac{0.050}{292}=\frac{V_2}{425}$; hence $V_2=7.3\times10^{-2}\text{ m}^3$.

12 The mass is just $28\times 2=56\text{ g}$.

The volume is found from $V=\frac{nRT}{P}=\frac{2.0\times 8.31\times 273}{1.0\times 10^5}=0.045\text{ m}^3$.

13 **a** $n=\frac{PV}{nRT}=\frac{2.35\times 10^5\times 1.25}{8.31\times 298}=141.3\text{ mol}$ and so we have

$N=nN_A=141.3\times 6.02\times 10^{23}=8.51\times 10^{25}$ molecules.

b In the final state of the gas we will have fewer molecules, a pressure of 2.05 atm, the same original volume and a temperature of 294 K.

Hence the final number of moles is $n=\frac{PV}{RT}=\frac{2.05\times 10^5\times 1.25}{8.31\times 294}=104.9\text{ mol}$.

This is a change of $141.3-104.9=36.4\text{ mol}$ and so

$N=nN_A=36.4\times 6.02\times 10^{23}=2.19\times 10^{25}$ escaped.

c The highest temperature is reached with the new, reduced number of moles and a pressure of 2.50 atm: $T=\frac{PV}{nR}=\frac{2.50\times 10^5\times 1.25}{104.9\times 8.31}=358\text{ K}$.

14 **a** From $\frac{P_1V_1}{n_1T_1}=\frac{P_2V_2}{n_2T_2}$ we deduce that $\frac{P_1}{T_1}=\frac{P_2}{T_2}$,

i.e. that $\frac{1.47}{413}=\frac{P_2}{488}$; hence $P_2=1.74\text{ atm}$.

b The original number of moles is $n_1=\frac{PV}{RT}=\frac{1.47\times 10^5\times 1.07}{8.31\times 413}=45.83\approx 46\text{ mol}$.

At the low temperature the number of moles is

$n_2=\frac{PV}{RT}=\frac{0.760\times 10^5\times 1.07}{8.31\times 319}=30.68\approx 31\text{ mol}$.

The number of **atoms** has not changed. There are $46\times N_A$ atoms in the container. At the low temperature there are n moles of the diatomic molecules and so $2n\times N_A$ atoms and $31-n$ moles of monatomic molecules and so $(31-n)\times N_A$ atoms. Thus $2n\times N_A+(31-n)\times N_A=46N_A$ and hence $n=15\text{ mol}$.

15 The molar mass of helium is 4.00 g per mole. A mass of 70.0 kg of helium corresponds to $\frac{70.0\times 10^3}{4.00}=1.75\times 10^4$ mol.

Thus $P=\frac{nRT}{V}=\frac{1.75\times 10^4\times 8.31\times 290}{404}=1.04\times 10^5$ Pa.

16 **a**

$$n=\frac{PV}{nRT}=\frac{150\times10^{3}\times5.0\times10^{-4}}{8.31\times300}=3.01\times10^{-2}\ \text{mol}$$

b

$$N=nN_A=3.01\times10^{-2}\times6.02\times10^{23}=1.8\times10^{22}$$

c

$$M=n\mu=3.01\times10^{-2}\times29=0.87\ \text{g}$$

17 **a**

$$V=\frac{nRT}{P}=\frac{1.0\times8.31\times273}{1.0\times10^{5}}=2.23\times10^{-2}\ \text{m}^{3}$$

b We have 1 mole and so 4.00 g of helium.
The density is thus

$$\rho=\frac{M}{V}=\frac{4.00\times10^{-3}}{2.23\times10^{-2}}=0.179\ \text{kg m}^{-3}$$

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c The change for oxygen is just the molar mass, and so

$$\rho=\frac{32}{4}\times0.179=1.43\ \text{kg m}^{-3}$$

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18 **a** One mole of helium at STP has a volume of

$$V=\frac{nRT}{P}=\frac{1.0\times8.31\times273}{1.0\times10^{5}}=2.23\times10^{-2}\ \text{m}^{3}.$$

This volume is shared by 6.02×10^{23} molecules and so the volume corresponding to one molecule is $v=\frac{2.23\times10^{-2}}{6.02\times10^{23}}=3.7\times10^{-26}\ \text{m}^{3}$.

One mole of water has mass 18 g and since the density is $1.00\times10^{3}\ \text{kg m}^{-3}$ the volume is $\frac{18\times10^{-3}}{1.00\times10^{3}}=1.8\times10^{-5}\ \text{m}^{3}$.

This volume is shared by 6.02×10^{23} molecules and so the volume corresponding to one molecule is $v=\frac{1.8\times10^{-5}}{6.02\times10^{23}}=3.0\times10^{-29}\ \text{m}^{3}$.

For uranium, one mole has mass 238 g and since the density is $18.7\times10^{3}\ \text{kg m}^{-3}$ the volume is $\frac{238\times10^{-3}}{18.7\times10^{3}}=1.27\times10^{-5}\ \text{m}^{3}$.

This volume is shared by 6.02×10^{23} molecules and so the volume corresponding to one molecule is $v=\frac{1.27\times10^{-5}}{6.02\times10^{23}}=2.1\times10^{-29}\ \text{m}^{3}$.

b Taking the cubic roots of these volumes we find,
for helium, $\sqrt[3]{3.7\times10^{-26}} = 3.3\times10^{-9}$ m ,
for water $\sqrt[3]{3.0\times10^{-29}} = 3.1\times10^{-9}$ m and
for uranium $\sqrt[3]{2.1\times10^{-29}} = 2.8\times10^{-10}$ m .

c These are quite close to the actual radii.

19 Under the given changes the volume will stay the same and so the density will be unchanged.