

Self-assessment answers: 9 Trigonometric equations and identities

1. $3x = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$ or $\frac{5\pi}{6} + 2k\pi$ for $k \in \mathbb{Z}$.

$$\Rightarrow 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \dots$$

$$\Rightarrow x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$$

[4 marks]

2. Using the identity $\sin^2 x = 1 - \cos^2 x$:

$$2\sin^2 x - \cos x = 1 \Rightarrow 2\cos^2 x + \cos x - 1 = 0$$

$$\Rightarrow (2\cos x - 1)(\cos x + 1) = 0$$

$$\Rightarrow \cos x = \frac{1}{2} \text{ or } -1$$

$$\Rightarrow x = \pm \frac{\pi}{3}, \pm\pi$$

[8 marks]

3. $\sin x (\sin x - \sqrt{3} \cos x) = 0$

$$\Rightarrow \sin x = 0 \text{ or } \tan x = \sqrt{3}$$

$$\Rightarrow x = 0^\circ, 60^\circ, 240^\circ, 360^\circ$$

[6 marks]

4. Using double angle formula, $\cos 2\theta = 1 - 2 \sin^2 \theta$

$$\Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$\Rightarrow (2 \sin \theta + 1)(\sin \theta - 1) = 0$$

$$\Rightarrow \sin \theta = -\frac{1}{2} \text{ or } 1$$

$$\Rightarrow \theta = 90^\circ, 210^\circ \text{ or } 330^\circ$$

[6 marks]