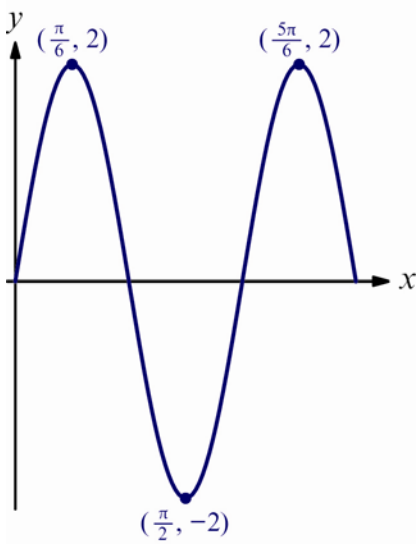
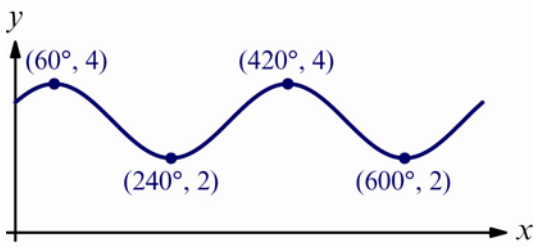
Self-assessment: 8 Circular measure and trigonometric functions

1. Find the constants a, b, c, d in the equations of the two graphs below:

(a) $y = a \sin(bx)$



(b) $y = \cos(x° - d°) + c$



(accessible to students on the path to grade 5 or 6) [7 marks]

2. Find the exact period of the function $f(x) = \sin 4x + \sin 6x$.

(accessible to students on the path to grade 3 or 4) [3 marks]

3. The depth of water in a harbour varies with time as $h = 8.6 + 1.2 \sin\left(\frac{\pi t}{6}\right)$, where h is the depth measured in metres and t is time in hours after midnight.

(a) Find the depth of the water at 2 p.m.

(b) What is the least depth of the water?

(c) At what times is the depth of the water 8.1 m?

(d) A ship can enter the harbour when the water depth is above 9 m. Find the times when the ship can enter the harbour.

(accessible to students on the path to grade 3 or 4) [8 marks]

4. Do not use a calculator to answer this question.

Let $f(x) = 3 \sin\left(x + \frac{\pi}{4}\right)$ for $x \in [0, 2\pi]$.

(a) Find the exact value of $f\left(\frac{\pi}{12}\right)$.

(accessible to students on the path to grade 5 or 6)

(b) Find the exact values of all the zeroes of f.

(c) State the minimum value of $5 - f(x)$.

Another function is defined by $g(x) = \tan\left(x - \frac{\pi}{4}\right)$ for $x \in [0, 2\pi]$.

(accessible to students on the path to grade 3 or 4)

(d) Find the exact value of $g\left(\frac{\pi}{2}\right)$.

(e) By sketching graphs, or otherwise, find the number of solutions of the equation $f(x) = g(x)$.

(accessible to students on the path to grade 5 or 6)

[12 marks]