

Self-assessment answers: 3 Algebraic structures

1. Let $y = x^2$

$$y^2 - 5y + 4 = 0$$

$$\Rightarrow (y - 1)(y - 4) = 0$$

$$\Rightarrow y = 1, 4$$

So, $x^2 = 1, 4$

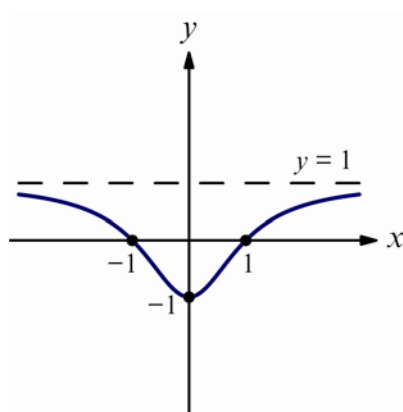
$$\Rightarrow x = \pm 1, \pm 2$$

[5 marks]

2. Using a GDC, $x = 0.241$ (3SF)

[4 marks]

3.



[4 marks]

4. $e^{2x} + 2e^x - 15 = 0$

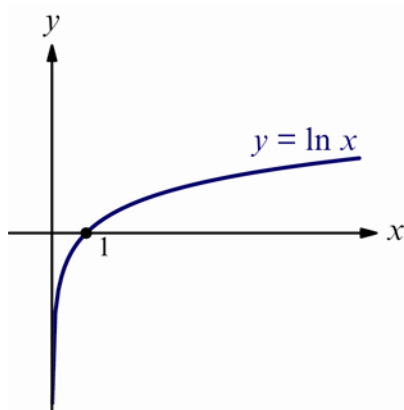
$$\Rightarrow (e^x + 5)(e^x - 3) = 0$$

$$\Rightarrow e^x = -5 \text{ or } 3. \text{ Reject the negative solution.}$$

$$\Rightarrow x = \ln 3$$

[5 marks]

5. (a)



[3 marks]

$$(b) \log_3(9x^2) - \log_3(6x) = \log_3\left(\frac{9x^2}{6x}\right)$$

$$= \log_3\left(\frac{3x}{2}\right)$$

$$= \log_3 3x - \log_3 2$$

$$= \log_3 x + \log_3 3 - \log_3 2$$

$$= \log_3 x - \log_3 2 + 1$$

[5 marks]

$$(c) \log_3(9x^2) - \log_3(6x) = 2x \log_3 x - \log_3 2 + 1$$

$$\Rightarrow \log_3 x - \log_3 2 + 1 = 2x \log_3 x - \log_3 2 + 1$$

$$\Rightarrow (1 - 2x) \log_3 x = 0$$

$$\Rightarrow 1 - 2x = 0 \text{ or } \log_3 x = 0$$

$$\Rightarrow x = \frac{1}{2}, 1$$

[4 marks]