

**Self-assessment answers: 2 Exponents and logarithms**

1. (a)  $\log(5^3 \div 20 \times 16) = \log\left(\frac{5 \times 5 \times 5 \times 4 \times 4}{5 \times 4}\right) = \log(5 \times 5 \times 4) = \log(100) = 2$  [2 marks]

(b)  $\ln\left(\frac{45}{4}\right) = \ln\left(\frac{5 \times 3^2}{2^2}\right) = \ln 5 + 2 \ln 3 - 2 \ln 2 = z + 2y - 2x$  [2 marks]

(c)  $\ln K = 2 - \ln c \Leftrightarrow K = 2^{2 - \ln c} = \frac{e^2}{e^{\ln c}} = \frac{e^2}{c}$  [2 marks]

2.  $\log_2\left(\frac{x+2}{x}\right) = 3 \Leftrightarrow \frac{x+2}{x} = 2^3 = 8 \Leftrightarrow x+2 = 8x \Leftrightarrow x = \frac{2}{7}$  [3 marks]

3. Substitute  $y = e^x$ :

$$3y^2 - 7y + 2 = 0$$

$$\Leftrightarrow (3y - 1)(y - 2) = 0$$

$$\Leftrightarrow y = \frac{1}{3} \text{ or } y = 2$$

$$\Leftrightarrow x = \ln\left(\frac{1}{3}\right) = -\ln 3 \text{ or } x = \ln 2$$
 [5marks]

4. (a)  $C = 2, 2 + A = 7 \Rightarrow A = 5$

(b)  $3 = 2 + 5e^{-k \times 2} \Leftrightarrow e^{-2k} = \frac{1}{5} \Leftrightarrow e^{2k} = 5 \Leftrightarrow k = \frac{\ln 5}{2}$  [5 marks]

5. (Note that exact values have been used in the calculations for parts (a) (iii), (b) (i) and (iii).)

(a) (i)  $A = 50$

(ii)  $t = 3, N = 270 \Rightarrow 270 = 50 \times b^{3k}$

$\Rightarrow b^{3k} = \frac{270}{50} = 5.4 \Rightarrow (b^k)^3 = 5.4 \Rightarrow b^k = \sqrt[3]{5.4} = 1.75$  (3SF)

(iii)  $N = 50 \times b^{5k} = 50 \times (b^k)^5 = 50 \times (1.75)^5 = 831$  (3SF) [6 marks]

(b) (i) When  $t = 5$ :  $2000 - Me^{-0.47 \times 5} = 831 \Rightarrow M = \frac{2000 - 831}{e^{-2.35}} = 12300$  (3SF)

(ii) When  $t$  is large,  $e^{-0.47t} \rightarrow 0$ , so  $N \rightarrow 2000$

(iii)  $1999 = 2000 - 12300e^{-0.47t} \Leftrightarrow e^{-0.47t} = \frac{1}{12300} \Rightarrow t = \frac{\ln\left(\frac{1}{12300}\right)}{-0.47} = 20$  minutes

(Part (iii) can also be done by drawing the graph on your GDC.) [5 marks]