Self-assessment: 2 Exponents and logarithms

1. Do not use a calculator to answer this question.

(a) Find the exact value of $3\log(5) - \log(20) + \log(16)$.

(b) Given that $x = \ln 2$, $y = \ln 3$ and $z = \ln 5$, express $\ln\left(\frac{45}{4}\right)$ in terms of x, y and z.

(c) If $\ln K = 2 - \ln c$, find and simplify an expression for K in terms of c.

(accessible to students on the path to grade 3 or 4) [6 marks]

2. Do not use a calculator to answer this question.

Solve the following equation:

$$\log_2(x+2) - \log_2 x = 3$$

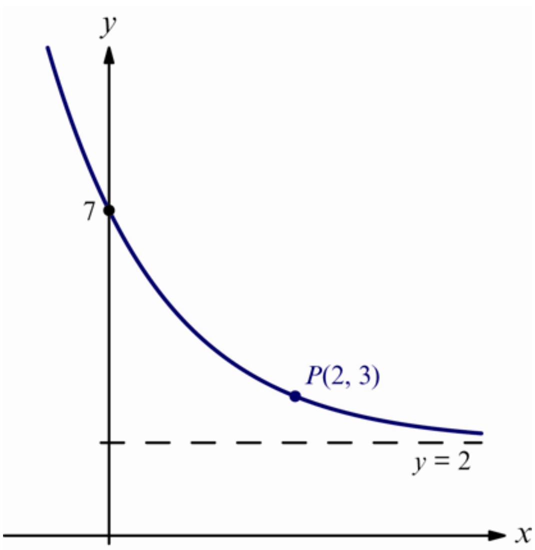
(accessible to students on the path to grade 3 or 4) [3 marks]

3. Find the exact solutions of the equation,

$$3e^{2x} - 7e^{x} + 2 = 0$$

(accessible to students on the path to grade 5 or 6) [5 marks]

4. The diagram shows the graph with equation $y = C + Ae^{-kt}$. The graph passes through the point P(2, 3).



(a) Write down the value of C and the value of A.

(b) Find the exact value of k.

(accessible to students on the path to grade 5 or 6) [5 marks]

5. (a) The population of bacteria increases according to an exponential model, $N = A \times b^{kt}$, where N is the number of bacteria after t minutes and A and b are positive constants. Given that initially there were 50 bacteria and that after three minutes the number has grown to 270,

(i) Write down the value of A.

(ii) Show that, to three significant figures, $b^k = 1.75$.

(iii) Find the size of the population after five minutes.

(b) After five minutes the population growth slows down, so that now it follows the new model,

$$N = 2000 - Me^{-0.47t}.$$

(i) Find the value of M.

(ii) According to this model, the size of the population approaches a limit in the long term. Find this limit.

(iii) How long does it take for the population size to reach1999? Give your answer to the nearest minute.

(accessible to students on the path to grade 5 or 6) [11 marks]

