

Self-assessment answers: 7 Sequences and series

1. (a) $u_{15} = 3 + 14 \times 0.7 = 12.8$ [1 mark]
- (b) $u_{10} = 10 \times 1.2^9 = 51.6$ (3SF) [1 mark]
- (c) $S_{20} = \frac{20}{2} (2 \times 1 + 19(-2.4)) = -436$ [1 mark]
- (d) $S_{\infty} = \frac{21}{1-0.8} = 105$ [1 mark]

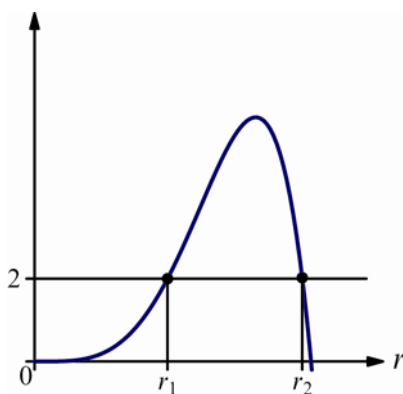
2.
$$\begin{cases} u_4 = a + 3d = 34 \\ u_{10} = a + 9d = 13 \end{cases}$$

$\Rightarrow a = 44.5, d = -3.5$ [4 marks]

3.
$$\begin{cases} u_5 = ar^4 = 2 \\ S_{\infty} = \frac{a}{(1-r)} = 72 \end{cases}$$

$a = 72(1 - r)$

$72(1 - r)r^4 = 2$



$\Rightarrow r = 0.481$ or 0.968 (from GDC) [5 marks]

4. $(2x + 3) - (x - 1) = (x^2 - 3) - (2x + 3)$

$$\Leftrightarrow x^2 - 3x - 10 = 0$$

$$\Leftrightarrow (x - 5)(x + 2) = 0$$

$$\therefore x = 5 \text{ or } -2$$

[4 marks]

5. (a) Arithmetic sequence, $u_1 = 10$, $d = 2$

[1 mark]

(i) $u_8 = 10 + 7 \times 2 = \text{£}24$

[1 mark]

(ii) $S_8 = \frac{8}{2}(20 + 7(2)) = \text{£}136$

[1 mark]

(iii) $S_n = \frac{n}{2}(20 + 2(n - 1)) = \frac{n}{2}(2n + 18)$

Using table (or graph) on GDC: $S_n > 200$ after 11 months

[3 marks]

(b) 1st month: 10

2nd month: $10 \times 1.01 + 10$

3rd month: $(10 \times 1.01 + 10) \times 1.01 + 10 = 10 \times 1.01^2 + 10 \times 1.01 + 10$

Geometric series, $u_1 = 10$, $r = 1.01$

[2 marks]

(i) $S_{10} = \frac{10(1.01^{10} - 1)}{1.01 - 1} = \text{£}104.62$

[1 mark]

(ii) $S_n = \frac{10(1.01^n - 1)}{1.01 - 1}$

$S_n > 200$ after 19 months

[2 marks]

(c) Freya: $F_n = \frac{10(1.01^n - 1)}{1.01 - 1}$

Caroline: $C_n = \frac{n}{2}(2n + 18)$

Using table (or graph) on GDC: $F_n > C_n$ after 590 months

[3 marks]