



Self-assessment: 25 Mathematical induction

1. Prove by induction that $1 \times 3 + 2 \times 4 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$.

(accessible to students on the path to grade 3 or 4) [6 marks]

2. Use the principle of mathematical induction to show that $15^n - 2^n$ is a multiple of 13 for all $n \in \mathbb{N}$.

(accessible to students on the path to grade 5 or 6) [6 marks]

3. Prove by induction that $3^n > n + 17$ for all integers $n \geq 3$.

(accessible to students on the path to grade 5 or 6) [6 marks]

4. (a) Use mathematical induction to prove that $(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$ for $n \in \mathbb{N}$.

(b) By expanding $(\cos \theta + i \sin \theta)^5$, find an expression for $\sin(5\theta)$ in terms of $\sin \theta$.

(accessible to students on the path to grade 5 or 6) [12 marks]