

Self-assessment answers: 23 Discrete probability distributions

1. (a) $\sum P(X = x) = 1$

$$\Rightarrow 3p + \frac{7}{10} = 1$$

$$\Rightarrow p = \frac{1}{10}$$

(b) $P(X \geq 4) = \frac{7}{10}$

(c) $E(X) = \sum xP(X = x) = \left(\frac{1}{10} + \frac{3}{5} + \frac{4}{10} + \frac{12}{10} + \frac{14}{5} \right) = \frac{51}{10}$ [8 marks]

2. (a) $\binom{6}{4} p^4 (1-p)^2 = 0.261$

$$\Rightarrow p = 0.529 \text{ or } 0.787 \text{ (GDC)}$$

(b) $P(X \leq 2 | p = 0.787) = 0.0213$

(c) $E(X) = np = 4.72$, $\text{Var}(X) = np(1-p) = 1.01$ [8 marks]

3. Let N be the number of postcards in a single week. $N \sim \text{Po}(5)$

(a) $P(N = 3) = 0.140$ (3SF)

(b) $\text{Var}(N) = 5$ so standard deviation = $\sqrt{5} = 2.24$ (3SF)

(c) Let F be the number of postcards in four weeks. $F \sim \text{Po}(20)$

$$\Rightarrow P(F < 15) = 0.105$$

(d) (i) $P(N > 5) = 1 - P(N \leq 5) = 0.384$

Probability of more than 5 in two weeks is $0.384^2 = 0.147$

(ii) Let X be the number of weeks in which more than 5 cards are received.

$$X \sim \text{B}(8, 0.384)$$

$$P(X = 4) = 0.219$$
 [14 marks]