# Self-assessment answers: 19 Further integration methods

**1.** $u = x - 1 \quad \Rightarrow \quad \mathrm{d}u = \mathrm{d}x$

$$\int x^2\sqrt{x-1}\,\mathrm{d}x = \int (u+1)^2\sqrt{u}\,\mathrm{d}u$$

$$= \int u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\,\mathrm{d}u$$

$$= \frac{2}{7}u^{\frac{7}{2}} + \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + c$$

$$= \left(\frac{2}{7}(x-1)^2 + \frac{4}{5}(x-1) + \frac{2}{3}\right)(x-1)^{\frac{3}{2}} + c$$

*[5 marks]*

**2.** Using integration by parts:

$$\int x\mathrm{e}^{-3x}\mathrm{d}x = -\frac{x}{3}\mathrm{e}^{-3x} + \frac{1}{3}\int \mathrm{e}^{-3x}\mathrm{d}x$$

$$= -\frac{\mathrm{e}^{-3x}}{9}(3x+1)$$

*[5 marks]*

**3.** $$\int_0^m \frac{\mathrm{d}x}{1+2x} = \left[\frac{1}{2}\ln(1+2x)\right]_0^m = \ln\sqrt{1+2m} = 3$$

$$\Rightarrow \quad m = \frac{1}{2}(\mathrm{e}^6 - 1)$$

*[5 marks]*

**4.** $\int_{-3}^{-4} \frac{1}{x^2+6x+10} \mathrm{d}x = \int_{-3}^{-4} \frac{1}{(x+3)^2+1} \mathrm{d}x$

Using substitution $u = x + 3$:

When $x = -4$, $u = -1$; when $x = -3$, $u = 0$; $\mathrm{d}u = \mathrm{d}x$

$\int_{-3}^{-4} \frac{1}{x^2+6x+10} \mathrm{d}x = \int_{0}^{-1} \frac{1}{u^2+1} \mathrm{d}u$

$= [\arctan u]_0^{-1}$

$= -\frac{\pi}{4}$ *[6 marks]*

**5.** (a) Let $u = \cos x$, then $\mathrm{d}u = -\sin x \, \mathrm{d}x$.

So, $\int \tan x \, \mathrm{d}x = \int \frac{\sin x}{\cos x} \mathrm{d}x = -\int \frac{1}{u} \mathrm{d}u$

$= -\ln|u| + c$

$= -\ln|\cos x| + c$

$= \ln|\sec x| + c$

(b) $\int_0^{\pi/3} \tan^3 x \, \mathrm{d}x = \int_0^{\pi/3} \tan x \left(\sec^2 x - 1\right) \mathrm{d}x$

$= \int_0^{\pi/3} \tan x \sec^2 x - \tan x \, \mathrm{d}x$

$= \left[\frac{1}{2}\sec^2 x - \ln|\sec x|\right]_0^{\pi/3}$

$= 2 - \frac{1}{2} - \ln 2$

$= \frac{3}{2} - \ln 2$ *[9 marks]*