

## Self-assessment: 15 Complex numbers

1. Solve the equation  $z + 2z^* = 4i - 1$ .

*(accessible to students on the path to grade 3 or 4) [5 marks]*

2. Do not use a calculator to answer this question.

Write  $\frac{\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)^4}{\cos \frac{5\pi}{12} - i \sin \frac{5\pi}{12}}$  in the form  $a + bi$ .

*(accessible to students on the path to grade 5 or 6) [5 marks]*

3. Do not use a calculator to answer this question.

Find all the roots of the equation  $z^3 = -4 + 4\sqrt{3}i$ . Give your answers in the form  $re^{i\theta}$ .

*(accessible to students on the path to grade 5 or 6) [8 marks]*

4. Given that  $z = \cos \theta + i \sin \theta$ ,

(a) Show that  $z^n - \frac{1}{z^n} = 2i \sin(n\theta)$ .

*(accessible to students on the path to grade 5 or 6)*

(b) Expand  $\left(z - \frac{1}{z}\right)^5$ .

*(accessible to students on the path to grade 3 or 4)*

(c) Hence, or otherwise, show that  $\sin^5 \theta = \frac{1}{16} \sin 5\theta - \frac{5}{16} \sin 3\theta + \frac{5}{8} \sin \theta$ .

*(accessible to students on the path to grade 7)*

*[12 marks]*