

Self-assessment answers: 12 Further trigonometry

1. Using double angle formula, $\cos 2\theta = 1 - 2 \sin^2 \theta$

$$\Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$\Rightarrow (2 \sin \theta + 1)(\sin \theta - 1) = 0$$

$$\Rightarrow \sin \theta = -\frac{1}{2} \text{ or } 1$$

$$\Rightarrow \theta = 90^\circ, 210^\circ \text{ or } 330^\circ$$

[6 marks]

2. Using the identity $1 + \cot^2 x = \csc^2 x$

$$\Rightarrow \csc^2 x - 3 \csc x + 2 = 0$$

$$\Rightarrow (\csc x - 2)(\csc x - 1) = 0$$

$$\Rightarrow \csc x = 2 \text{ or } 1$$

$$\Rightarrow x = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$$

[8 marks]

3. (a) Express height of top of picture above horizontal dashed line in terms of x in two different ways, then use the compound-angle identity $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$.

$$3 \tan x^\circ + 2 = 3 \tan(x^\circ + 30^\circ)$$

$$\Rightarrow 3 \tan x^\circ + 2 = 3 \frac{\tan x^\circ + 1/\sqrt{3}}{1 - \tan x^\circ/\sqrt{3}} = 3 \frac{\sqrt{3} \tan x^\circ + 1}{\sqrt{3} - \tan x^\circ}$$

$$\Rightarrow 3\sqrt{3} \tan x^\circ + 2\sqrt{3} - 3 \tan^2 x^\circ - 2 \tan x^\circ = 3\sqrt{3} \tan x^\circ + 3$$

$$\Rightarrow 3 \tan^2 x^\circ + 2 \tan x^\circ + (3 - 2\sqrt{3}) = 0 \text{ as required.}$$

(b) From calculator, $x^\circ = 10.3^\circ$

[9 marks]

4. (a) x

(b) $\cos(2 \arccos x) = \cos^2(\arccos x) - 1$
 $= x^2 - 1$

(c) $x^2 - 1 = x$

$$\Rightarrow x^2 - x - 1 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$$

Reject $\frac{1 + \sqrt{5}}{2}$ as greater than 1.

$$\Rightarrow x = \frac{1 - \sqrt{5}}{2}$$

[7 marks]